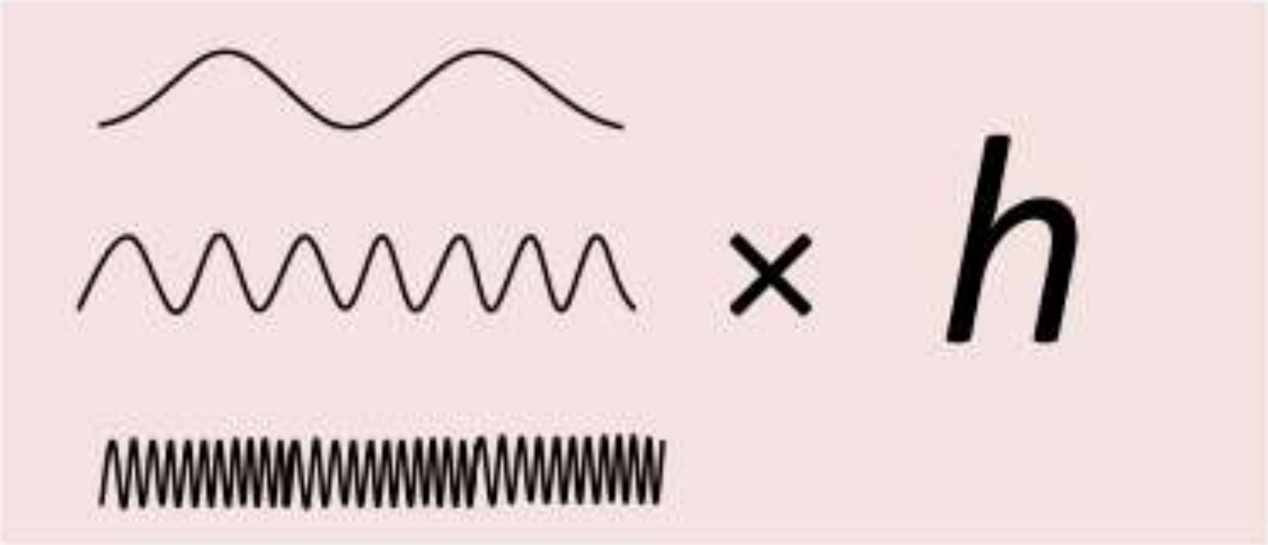
Wave function and Quantum Physics

The Planck Constant: *h*

$h = 6.626 \times 10^{-34}$ J s

a proportionality between frequency (ν) and energy

$$E = h\nu$$



Properties of matter

- Consists of discreet particles
  - Atoms, Molecules etc.
- Matter has momentum (mass)
- A well defined trajectory
- Does not diffract or interfere
  - 1 particle + 1 particle = 2 particles

The Compton Effect, Introduction

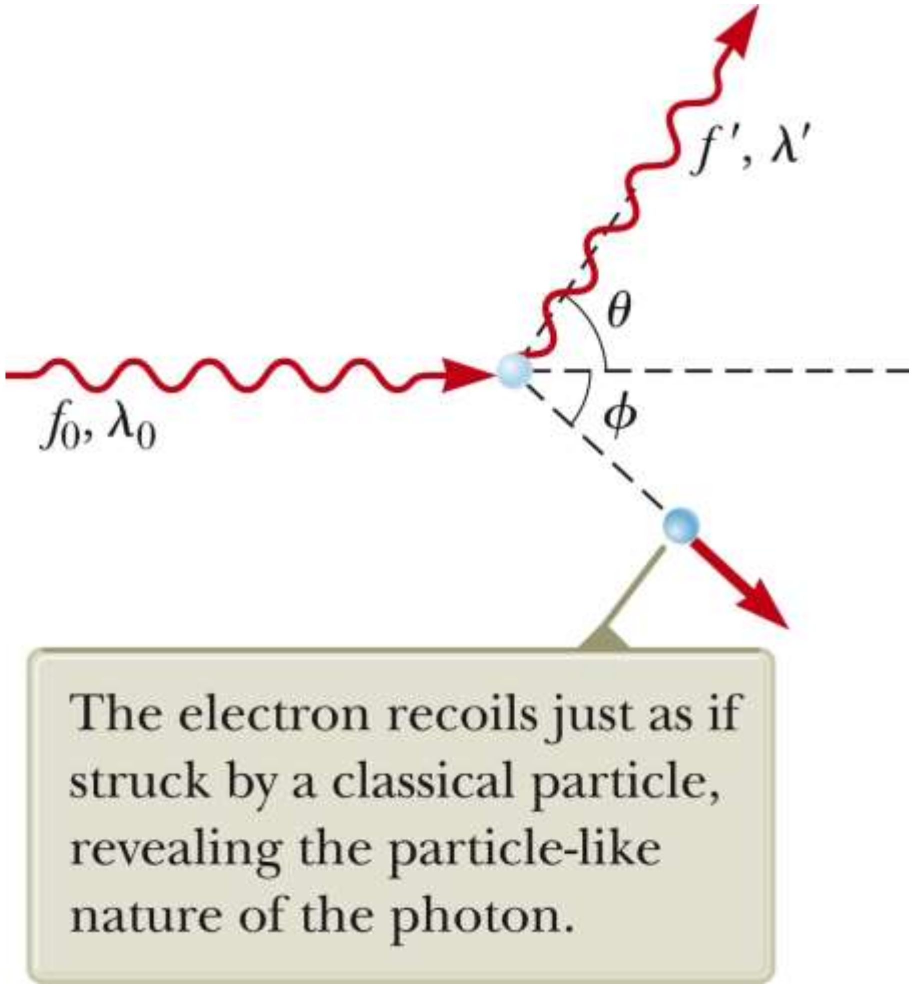
- Compton and Debye extended Einstein's idea of photon momentum.
- The two groups of experimenters accumulated evidence of the inadequacy of the classical wave theory.
- The classical wave theory of light failed to explain the scattering of x-rays from electrons.

Compton Effect, Classical Predictions

- According to the classical theory, em waves incident on electrons should:
  - Have radiation pressure that should cause the electrons to accelerate
  - Set the electrons oscillating
    - There should be a range of frequencies for the scattered electrons.

Compton Effect, Observations

- Compton's experiments showed that, at any given angle, only *one* frequency of radiation is observed.



Compton Effect, Explanation

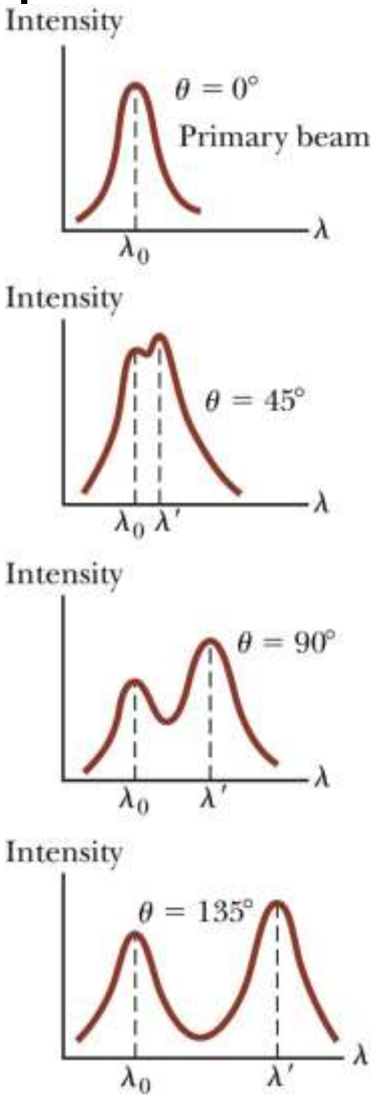
- The results could be explained by treating the photons as point-like particles having energy *hf* and momentum *h f / c.*
- Assume the energy and momentum of the isolated system of the colliding photon-electron are conserved.
- This scattering phenomena is known as the **Compton effect.**

Compton Shift Equation

- The graphs show the scattered x-rays for various angles.
- The shifted peak, *λ’*, is caused by the scattering of free electrons.

$$\lambda' - \lambda_o = \frac{h}{m_e c}(1 - \cos\theta)$$

- This is called the **Compton shift equation.**



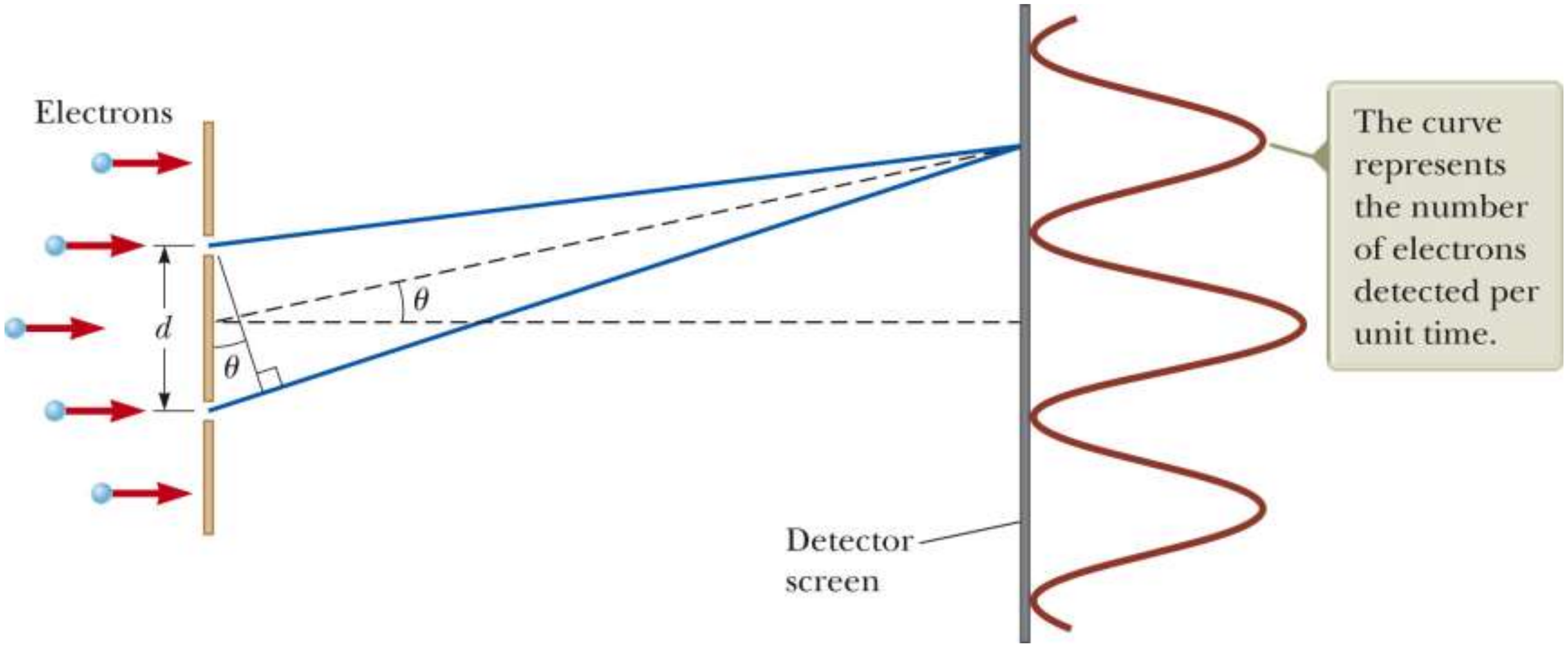
Compton Wavelength

- The factor $h/m_e c$ in the equation is called the **Compton wavelength** of the electron and is

$$\lambda_C = \frac{h}{m_e c} = 0.002\ 43\,\text{nm}$$

- The unshifted wavelength, λ_o, is caused by x-rays scattered from the electrons that are tightly bound to the target atoms.

Electron Diffraction, Set-Up

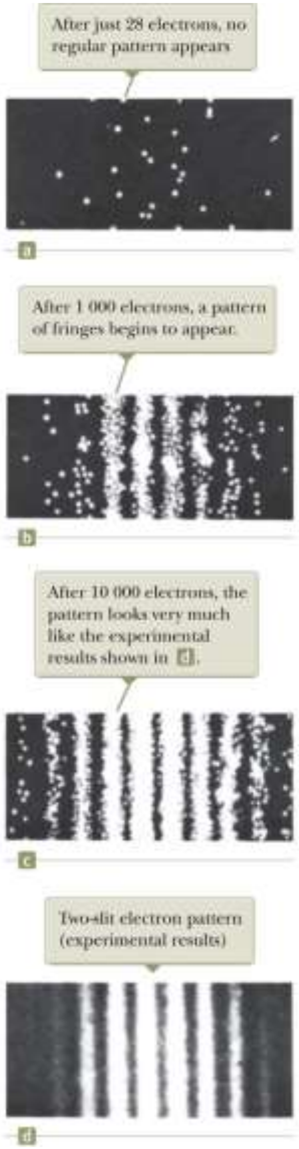


Electron Diffraction, Experiment

- Parallel beams of mono-energetic electrons that are incident on a double slit.
- The slit widths are small compared to the electron wavelength.
- An electron detector is positioned far from the slits at a distance much greater than the slit separation.

Electron Diffraction, cont.

- If the detector collects electrons for a long enough time, a typical wave interference pattern is produced.
- This is distinct evidence that electrons are interfering, a wave-like behavior.
- The interference pattern becomes clearer as the number of electrons reaching the screen increases.



Electron Diffraction, Equations

$$d \sin\theta = m\lambda$$

- A maximum occurs when
  - This is the same equation that was used for light.
- This shows the dual nature of the electron.
  - The electrons are detected as particles at a localized spot at some instant of time.
  - The probability of arrival at that spot is determined by finding the intensity of two interfering waves.

Electron Diffraction Explained

- An electron interacts with both slits simultaneously.
- If an attempt is made to determine experimentally which slit the electron goes through, the act of measuring destroys the interference pattern.
  - It is impossible to determine which slit the electron goes through.
- In effect, the electron goes through both slits.
  - The wave components of the electron are present at both slits at the same time.

Photons and Waves Revisited

- Some experiments are best explained by the photon model.
- Some are best explained by the wave model.
- We must accept both models and admit that the true nature of light is not describable in terms of any single classical model.
- The particle model and the wave model of light complement each other.
- A complete understanding of the observed behavior of light can be attained only if the two models are combined in a complementary matter.

Louis de Broglie

- 1892 – 1987
- French physicist
- Originally studied history
- Was awarded the Nobel Prize in 1929 for his prediction of the wave nature of electrons

Wave Properties of Particles

- Louis de Broglie postulated that because photons have both wave and particle characteristics, perhaps all forms of matter have both properties.

$$\lambda = \frac{h}{p} = \frac{h}{mu}$$

- The **de Broglie wavelength** of a particle is

Frequency of a Particle

- In an analogy with photons, de Broglie postulated that a particle would also have a frequency associated with it

$$f = \frac{E}{h}$$

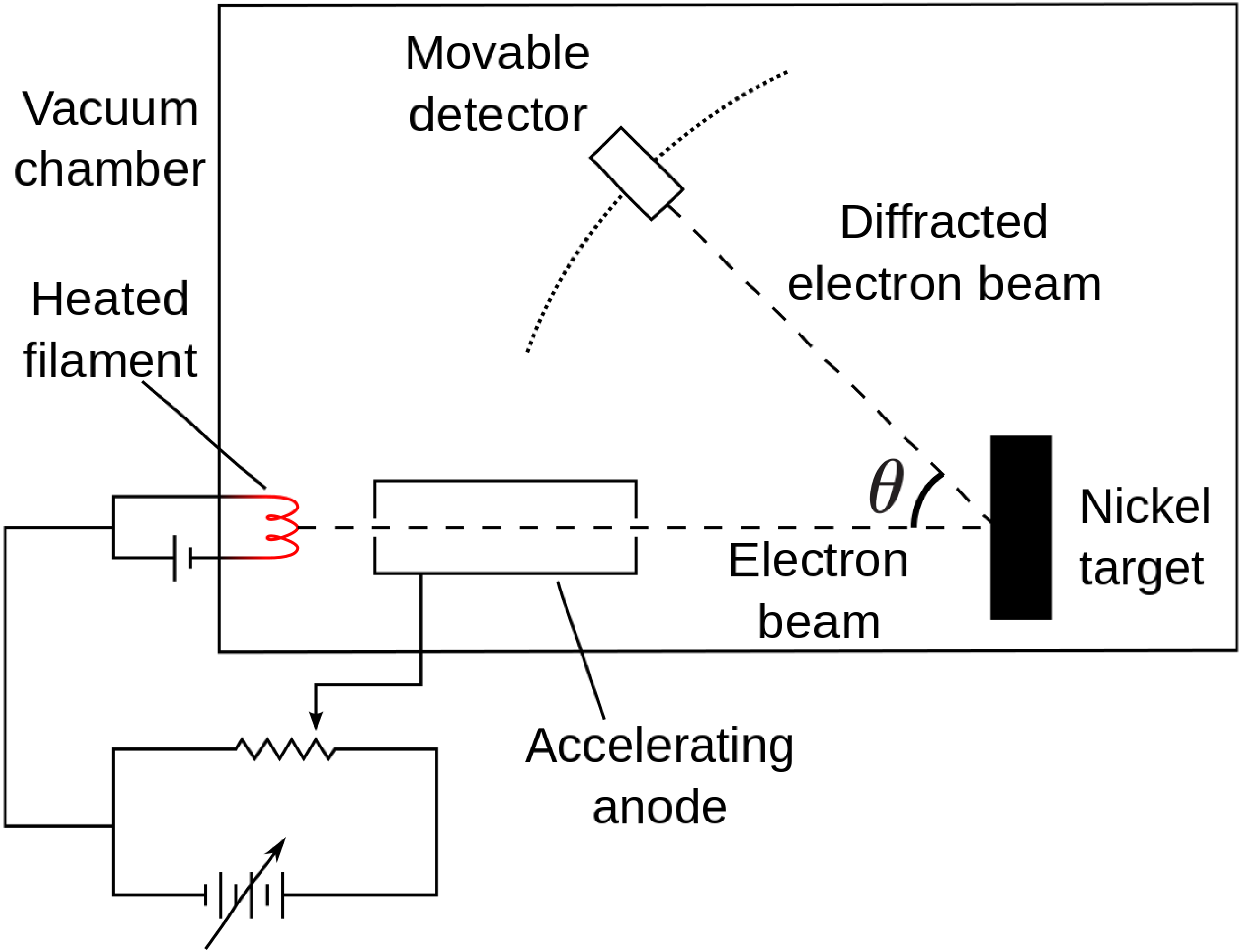
- These equations present the dual nature of matter:
  - Particle nature, p and E
  - Wave nature, λ and f

Complementarity

- The **principle of complementarity** states that the wave and particle models of either matter or radiation complement each other.
- Neither model can be used exclusively to describe matter or radiation adequately.

Davisson-Germer Experiment

- If particles have a wave nature, then under the correct conditions, they should exhibit diffraction effects.
- Davisson and Germer measured the wavelength of electrons.
- This provided experimental confirmation of the matter waves proposed by de Broglie.



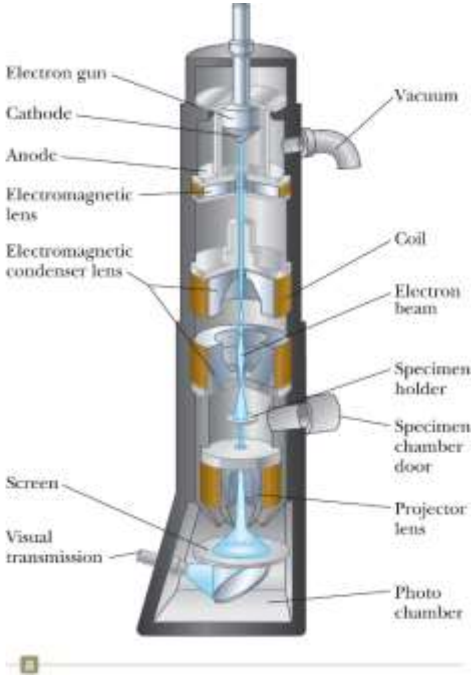
Vacuum chamber
Movable detector
Diffracted electron beam
Heated filament
θ
Nickel target
Electron beam
Accelerating anode

Wave Properties of Particles

- Mechanical waves have materials that are “waving” and can be described in terms of physical variables.
  - A string may be vibrating.
  - Sound waves are produced by molecules of a material vibrating.
  - Electromagnetic waves are associated with electric and magnetic fields.
- Waves associated with particles cannot be associated with a physical variable.

Electron Microscope

- The electron microscope relies on the wave characteristics of electrons.
- Shown is a *transmission* electron microscope
    - Used for viewing flat, thin samples
- The electron microscope has a high resolving power because it has a very short wavelength.
- Typically, the wavelengths of the electrons are about 100 times shorter than that of visible light.



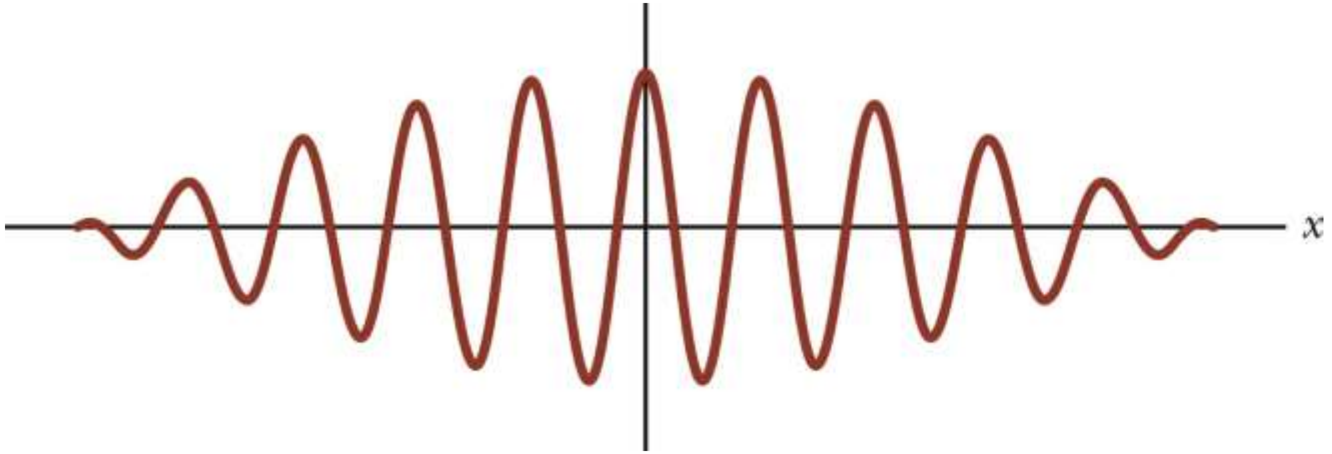
Quantum Particle

- The **quantum particle** is a new model that is a result of the recognition of the dual nature of both light and material particles.
- Entities have both particle and wave characteristics.
- We must choose one appropriate behavior in order to understand a particular phenomenon.

Ideal Particle vs. Ideal Wave

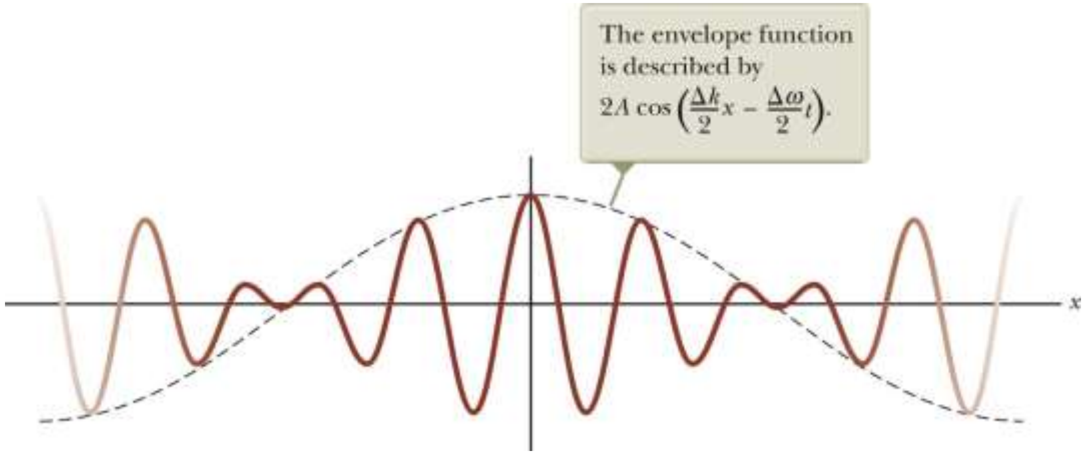
- An ideal particle has zero size.
  - Therefore, it is *localized* in space.
- An ideal wave has a single frequency and is infinitely long.
  - Therefore, it is *unlocalized* in space.
- A localized entity can be built from infinitely long waves.

Particle as a Wave Packet



- Multiple waves are superimposed so that one of its crests is at x = 0.
- The result is that all the waves add constructively at x = 0.
- There is destructive interference at every point except x = 0.
- The small region of constructive interference is called a wave packet.
  - The wave packet can be identified as a particle.

Wave Envelope



- The dashed line represents the envelope function.
- This envelope can travel through space with a different speed than the individual waves.

Speeds Associated with Wave Packet

- The **phase speed** of a wave in a wave packet is given by

$$v_{phase} = \omega / k$$

  - This is the rate of advance of a crest on a single wave.

- The group speed is given by

$$v_g = d\omega / dk$$

  - This is the speed of the wave packet itself.

Speeds, cont.

- The group speed can also be expressed in terms of energy and momentum.

$$v_g = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{1}{2m}(2p) = u$$

- This indicates that the group speed of the wave packet is identical to the speed of the particle that it is modeled to represent.

Werner Heisenberg

- 1901 – 1976
- German physicist
- Developed matrix mechanics
- Many contributions include:
  - Uncertainty principle
    - Received Nobel Prize in 1932
  - Prediction of two forms of molecular hydrogen
  - Theoretical models of the nucleus

The Uncertainty Principle

- In classical mechanics, it is possible, in principle, to make measurements with arbitrarily small uncertainty.
- Quantum theory predicts that it is fundamentally impossible to make simultaneous measurements of a particle's position and momentum with infinite accuracy.
- The **Heisenberg uncertainty principle** states: if a measurement of the position of a particle is made with uncertainty Δx and a simultaneous measurement of its x component of momentum is made with uncertainty Δp_x, the product of the two uncertainties can never be smaller than h/2

$$\Delta x \Delta p_x \geq \frac{\hbar}{2}$$

Heisenberg Uncertainty Principle, Explained

- It is physically impossible to measure simultaneously the exact position and exact momentum of a particle.
- The inescapable uncertainties do not arise from imperfections in practical measuring instruments.
- The uncertainties arise from the quantum structure of matter.

Heisenberg Uncertainty Principle, Another Form

- Another form of the uncertainty principle can be expressed in terms of energy and time.

$$\Delta E \Delta t \geq \frac{\hbar}{2}$$

- This suggests that energy conservation can appear to be violated by an amount ΔE as long as it is only for a short time interval Δt.

The Uncertainty Principle

- Classical physics
  - Measurement uncertainty is due to limitations of the measurement apparatus
  - There is no limit in principle to how accurate a measurement can be made
- Quantum Mechanics
  - There is a fundamental limit to the accuracy of a measurement determined by the Heisenberg uncertainty principle
  - If a measurement of position is made with precision Δx and a simultaneous measurement of linear momentum is made with precision Δp, then the product of the two uncertainties can never be less than $h/2\pi$

$$\Delta x \Delta p_x \geq \hbar$$

Uncertainty Principle, final

- The Uncertainty Principle cannot be interpreted as meaning that a measurement interferes with the system.
- The Uncertainty Principle is independent of the measurement process.
- It is based on the wave nature of matter.

Time-dependent Schrödinger equation:

we postulate the existence of a function of the coordinates called the wave function (State function), ψ. For one particle, one-dimensional system:

$$\psi = \psi(x, t)$$

It contains all information about a system. The probability of finding a particle in a given place can be given by ψ (Probability description).

Born postulates $|\psi(x,t)|^2 dx$ is the probability of finding a particle at position x and at time t (Probability density).
ψ must satisfy Schrödinger equation. As t passes, ψ changes to differential equation:

$$\frac{-\hbar}{i}\frac{\partial\Psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\ \Psi(x,t)}{\partial x^2} + V(x,t)\Psi(x,t)$$

Erwin Schrödinger
Nobel prize 1933

where, i= , m = particle mass, V(x,t) = potential energy.
This is called ***Time-dependent Schrödinger equation***.

Schrodinger Equation

Time independent Schrodinger equation

General form:

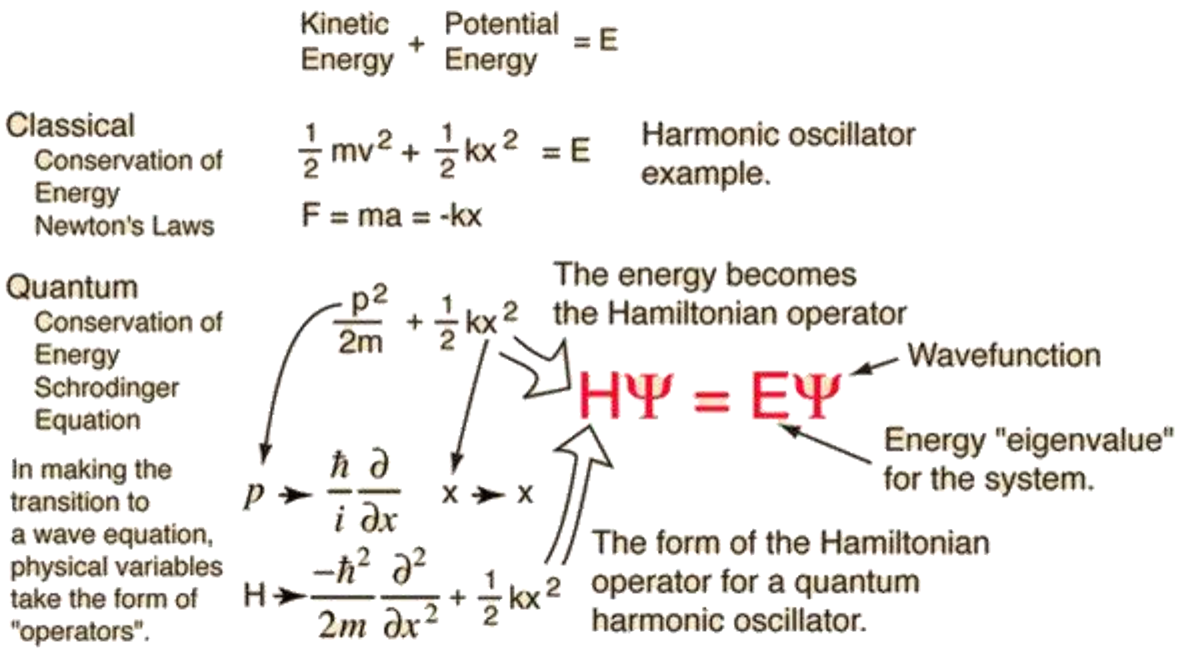
$$H\Psi = E\,\Psi$$

$$E = T + V$$

$$-j\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V(x)\psi$$

$$\hat{H}\psi = E\psi$$

The Schrodinger equation was discovered in 1926 by Erwin Schrodinger, an Austrian theoretical physicist.
It is an important equation that is fundamental to quantum mechanics.



$$\frac{-\hbar^2}{2m}\frac{\partial^2 u(x)}{\partial x^2} + V(x)u(x) = E\,u(x)$$

Quantum Theory

- Particles act like waves?!
- The best we can do is predict the probability that something will happen.

Heisenberg   Dirac   Schrodinger

Quantum mechanics

- Wave-particle duality
  - Waves and particles have interchangeable properties
  - This is an example of a system with complementary properties
- The mechanics for dealing with systems when these properties become important is called "Quantum Mechanics"