Mahidol University
Faculty of Science

Lecture 3.
Rotational motion and Oscillation

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Angular Position, Velocity and Acceleration: Life Science applications

Recall last t ime.

Rigid Body

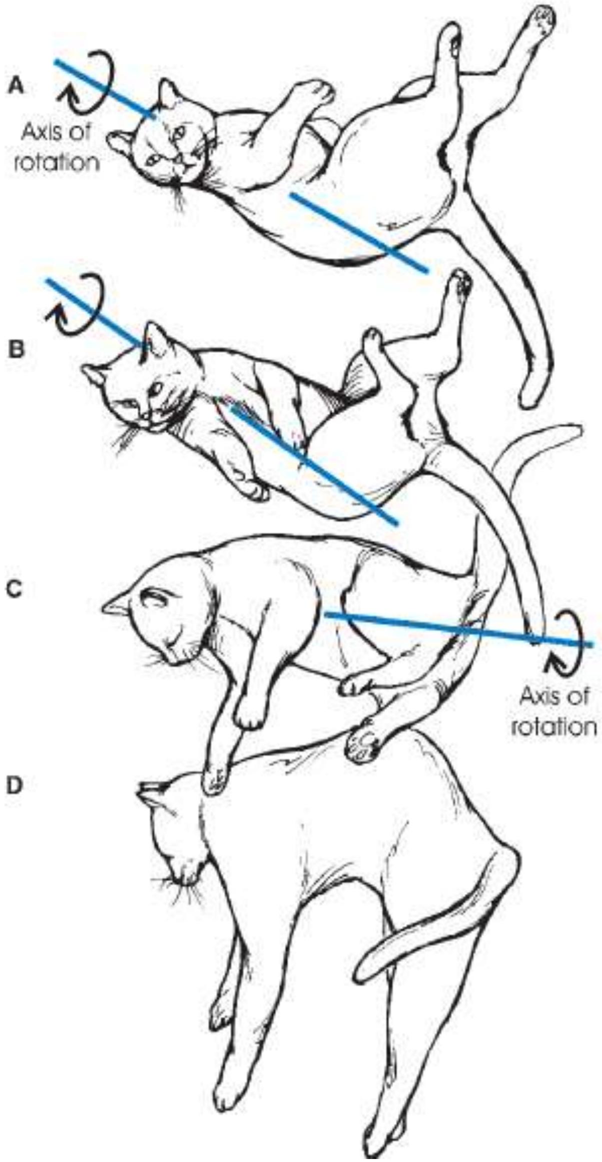
- An object or system of particles in which distances between the particles are constant. There is no deformity in the object.

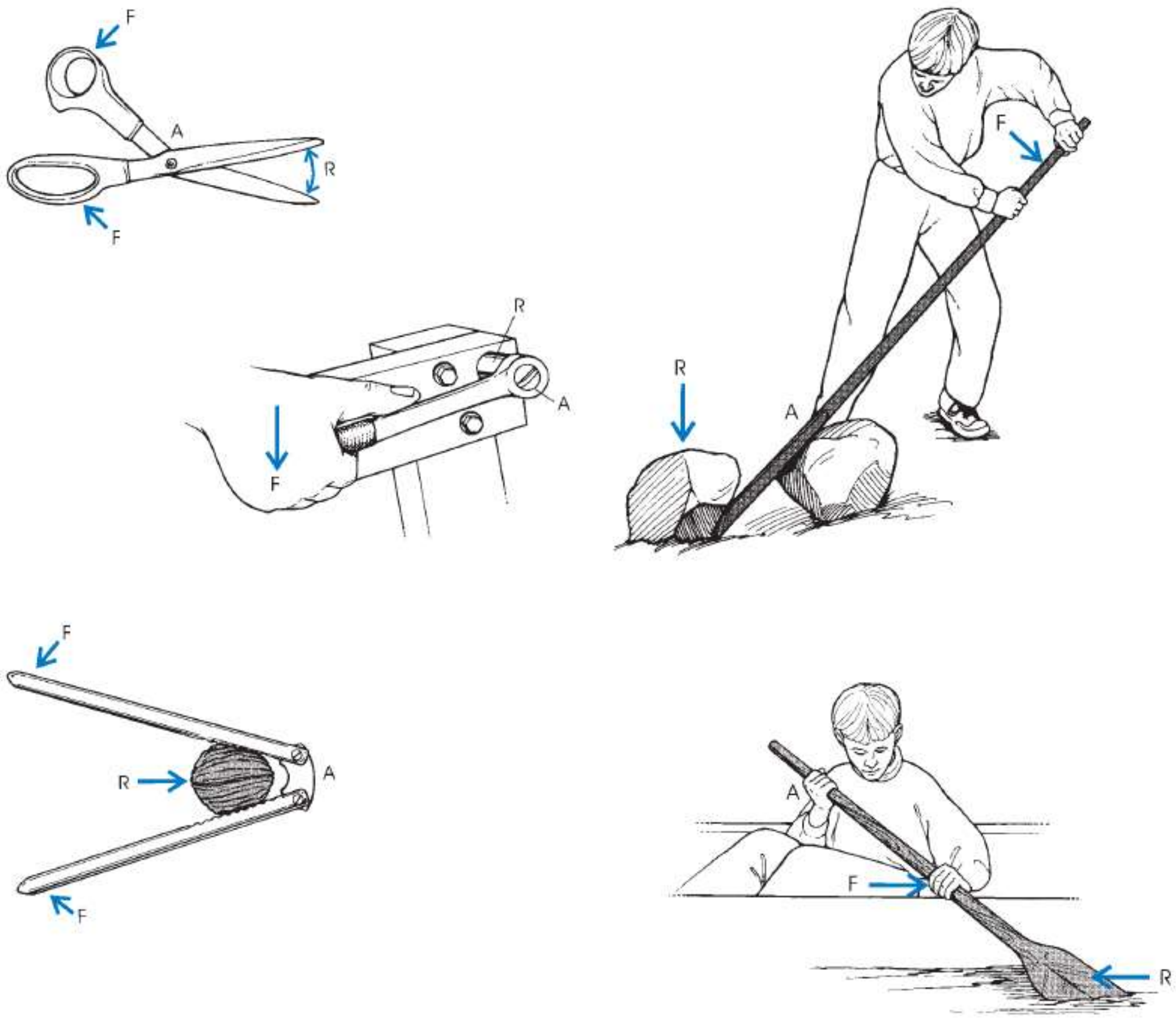
- Real object have some deformity.

Torque

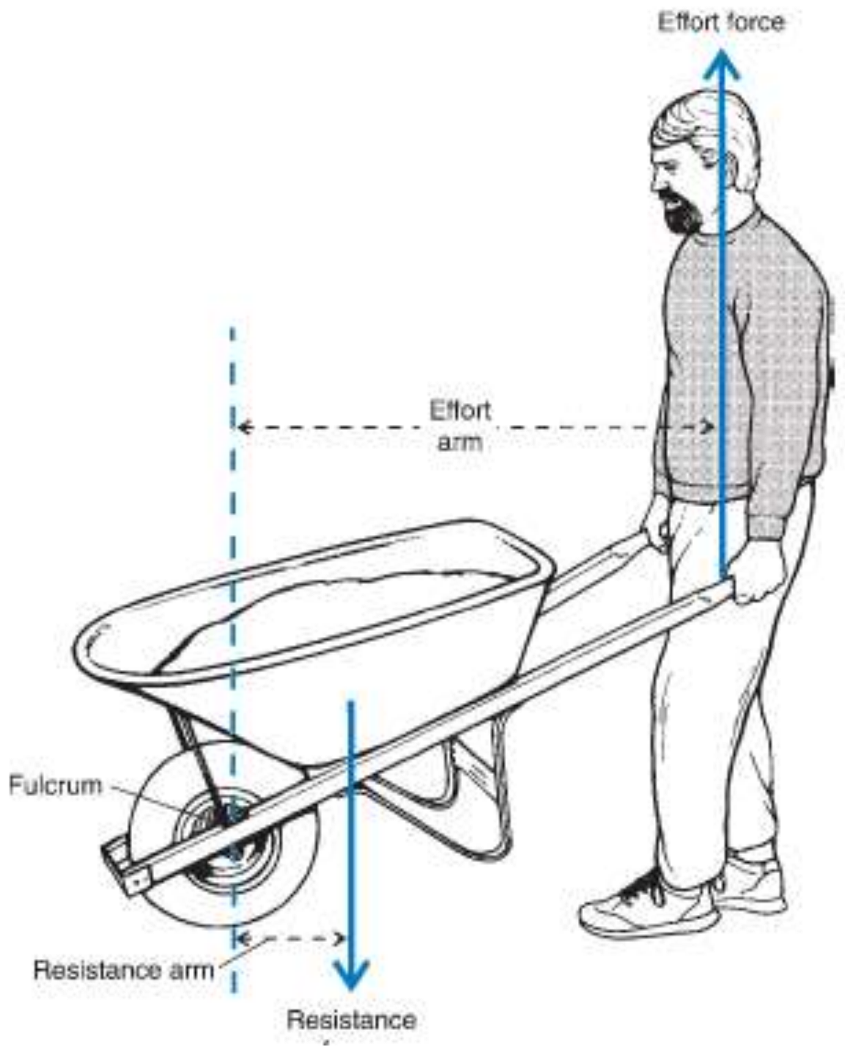
- The tendency of a force to cause rotation is called **torque.**

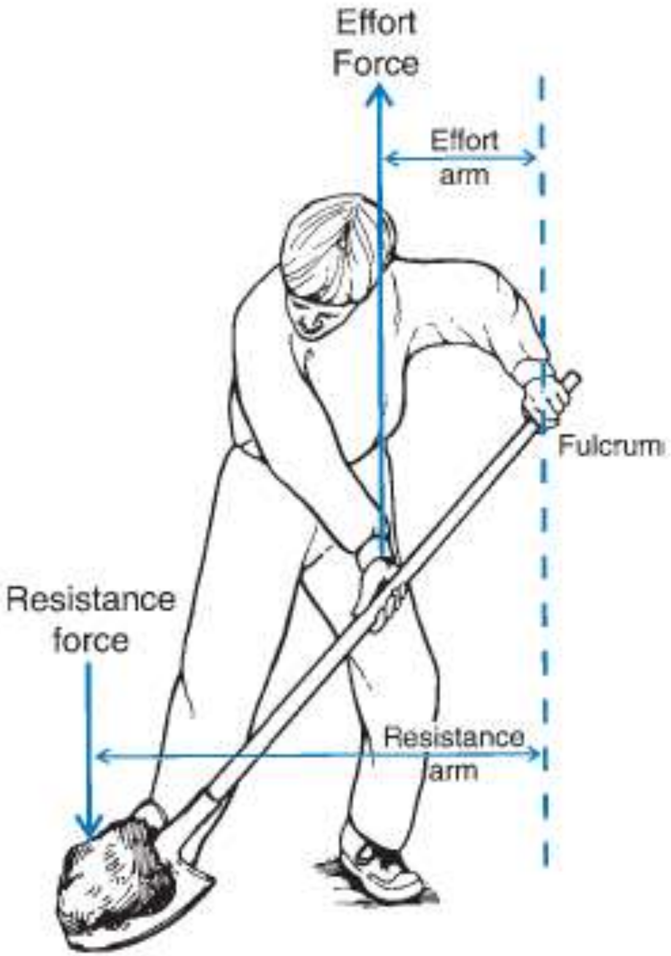
- Torque depends upon three factors:
  - Magnitude of the force
  - The direction in which it acts
  - The point at which it is applied on the object





Levers.


Effort force
Effort
arm
Fulcrum
Resistance arm
Resistance


Effort
Force
Effort
arm
Fulcrum
Resistance
force
Resistance
arm

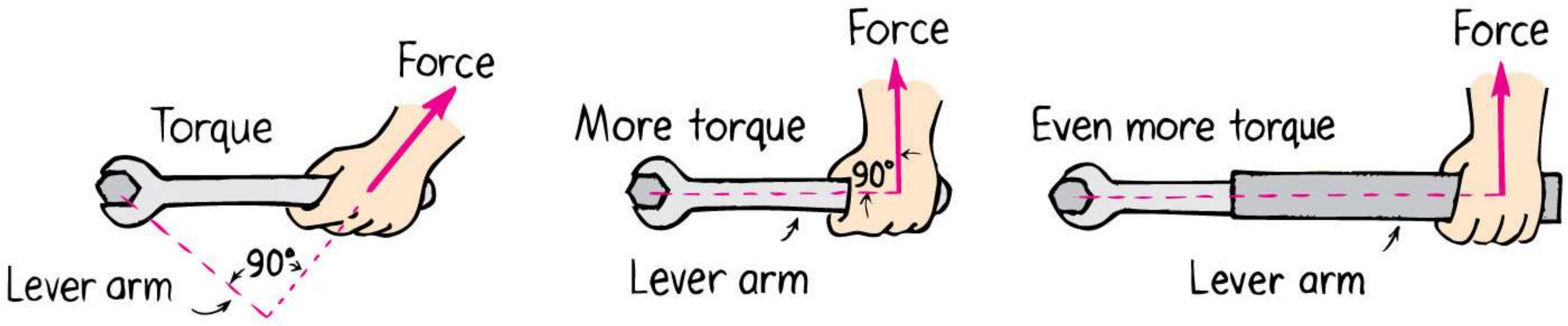
- The equation for Torque is

$$\text{Torque} = \text{lever arm} \times \text{force}$$

- The lever arm depends upon
  - where the force is applied.
  - the direction in which it acts.

Torque—Example

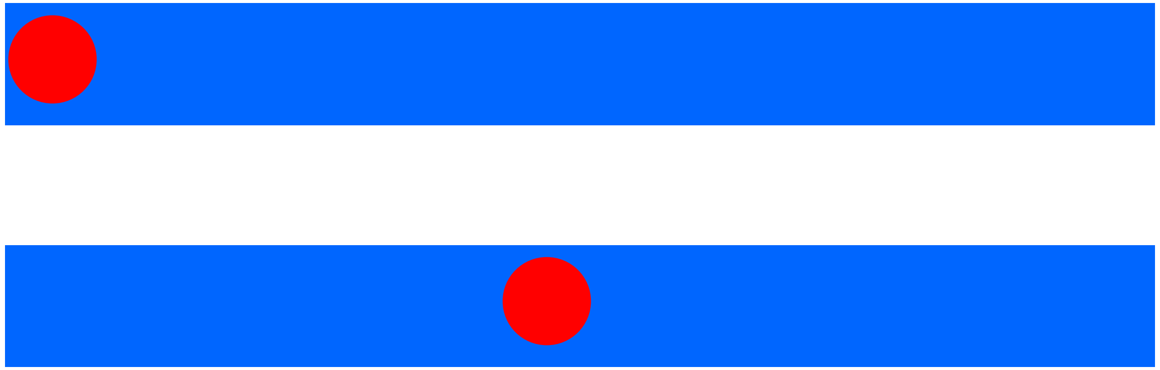
- 1st picture: Lever arm is *less than* length of handle because of direction of force.
- 2nd picture: Lever arm is equal to length of handle.
- 3rd picture: Lever arm is longer than length of handle.



The shape of an object determines how easy or hard it is to spin

For objects of the **same mass**,
the longer one is tougher to spin → takes more torque

It matters where the hinge is



The stick with the hinge at the end *takes more torque* to get it spinning than the stick with the hinge in the center.

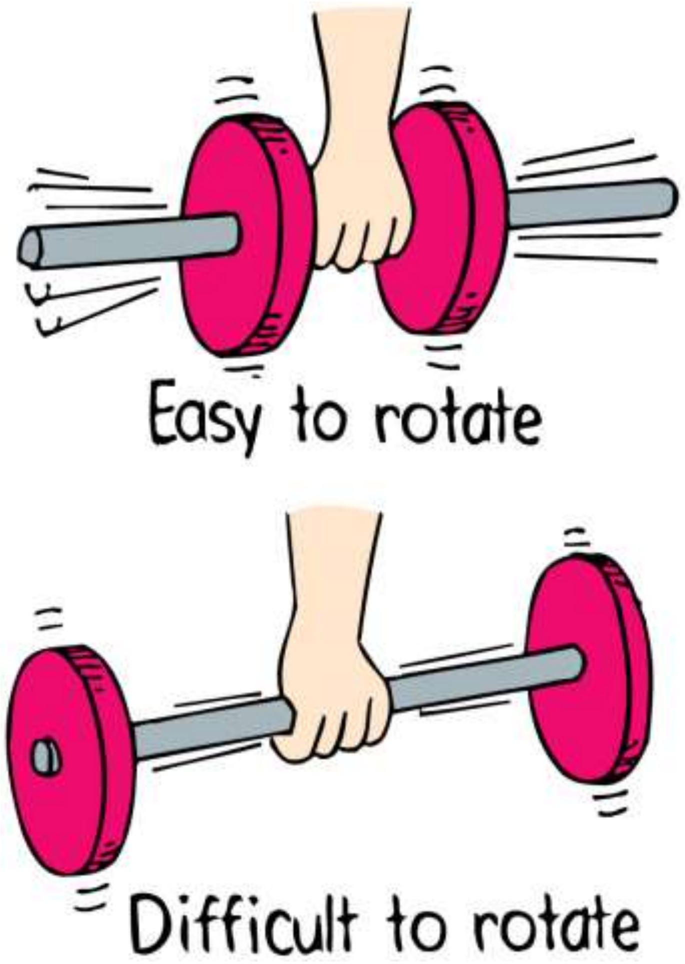
Rotational Inertia

- An object rotating about an axis tends to remain rotating about the same axis at the same rotational speed unless interfered with by some external influence.
- The property of an object to resist changes in its rotational state of motion is called **rotational inertia (symbol *I*).**

Rotational Inertia

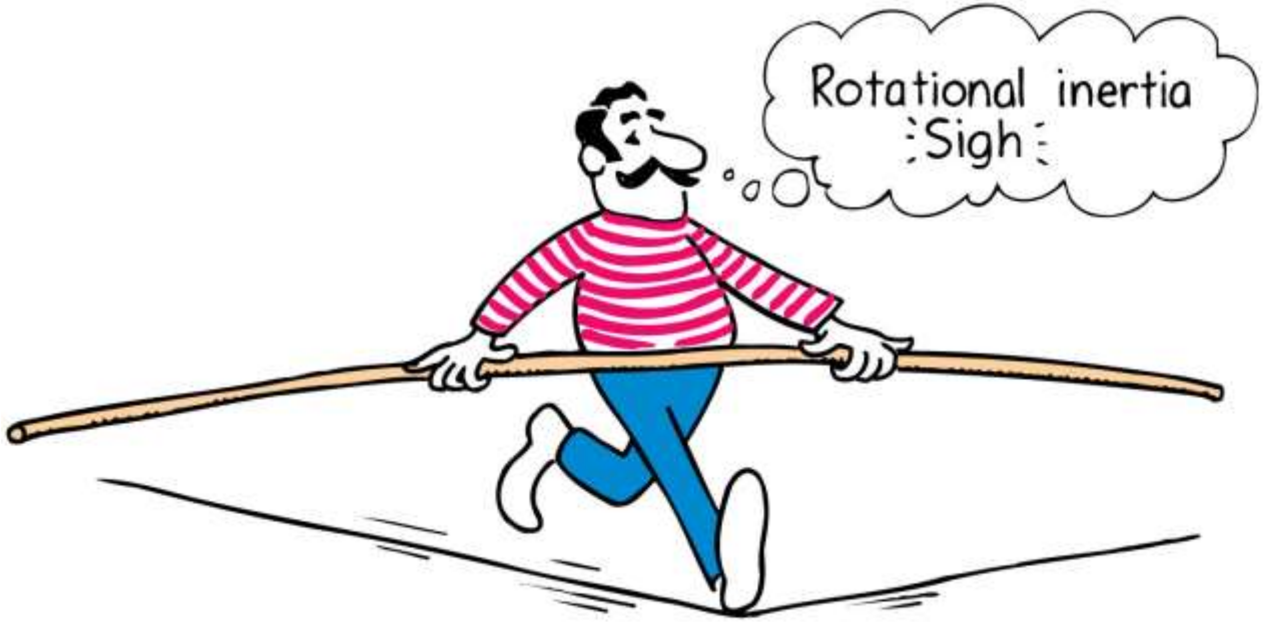
Depends upon

- mass of object.
- distribution of mass around axis of rotation.
  - The greater the distance between an object's mass concentration and the axis, the greater the rotational inertia.



Rotational Inertia

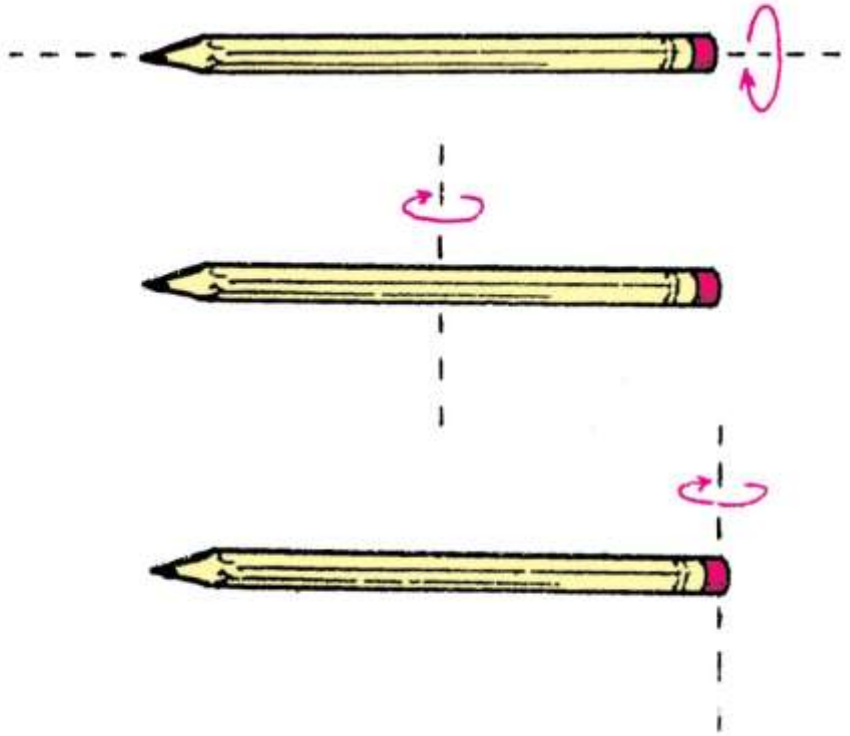
- The greater the rotational inertia, the harder it is to change its rotational state.
  - A tightrope walker carries a long pole that has a high rotational inertia, so it does not easily rotate.
  - Keeps the tightrope walker stable.



Rotational Inertia

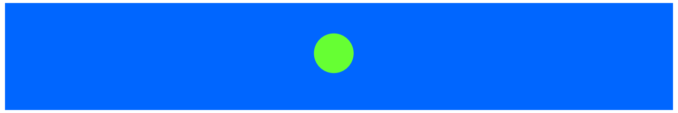
Depends upon the axis around which it rotates

- Easier to rotate pencil around an axis passing through it.
- Harder to rotate it around vertical axis passing through center.
- Hardest to rotate it around vertical axis passing through the end.



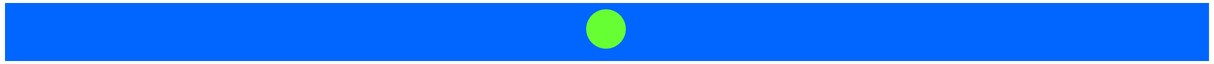
rotational inertia - examples

Suppose we have a rod of mass 2 kg and length 1 meter with the axis through the center



Its moment of inertia is 2 units

Imagine now that we take the same rod and stretch it out to 2 meters; its mass is, of course, the same.

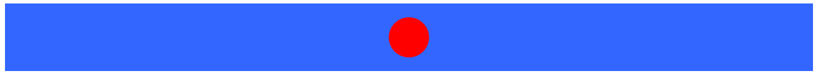


Its moment of inertia is 4 units

rotational inertia examples

Rods of equal mass and length

axes through center



Rotational inertia of 1 unit

axes through end

Rotational inertia of 4 units

Rotational Inertia
(moment of inertia)

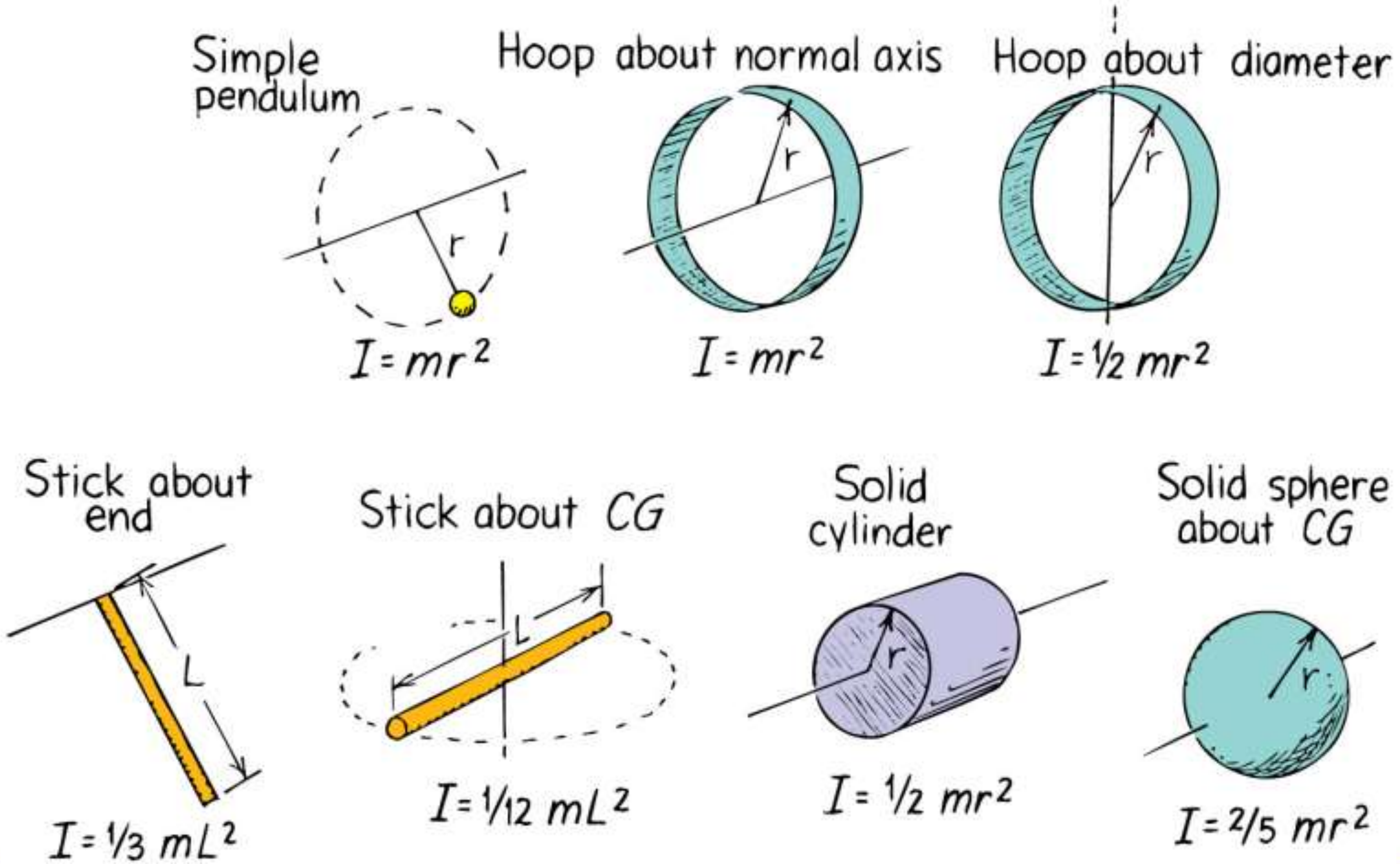
- **Rotational inertia** is a parameter that is used to quantify how much torque it takes to get a particular object rotating
- it depends not only on the mass of the object, but where the mass is relative to the hinge or axis of rotation
- the rotational inertia is bigger, if more mass is located farther from the axis.

How fast does it spin?

- For spinning or rotational motion, the rotational inertia of an object plays the same role as ordinary mass for simple motion
- For a given amount of torque applied to an object, its rotational inertia determines its rotational acceleration →
  the smaller the rotational inertia, the bigger the rotational acceleration

Rotational Inertia

The rotational inertia depends upon the shape of the object and its rotational axis.



Inertia is the resistance of any physical object to any change in its position and state of motion. This includes changes to the object's speed, direction, or state of rest.

Inertia is also defined as the tendency of objects to keep moving in a straight line at a constant velocity. The principle of inertia is one of the fundamental principles in classical physics that are still used to describe the motion of objects and how they are affected by the applied forces on them.

Inertia comes from the Latin word, *iners*, meaning idle, sluggish. Inertia is one of the primary manifestations of mass, which is a quantitative property of physical systems. Isaac Newton defined inertia as his first law in his *Philosophiæ Naturalis Principia Mathematica*, which states: [1]

The *vis insita*, or innate force of matter, is a power of resisting by which every body, as much as in it lies, endeavours to preserve its present state, whether it be of rest or of moving uniformly forward in a straight line.

https://en.wikipedia.org/wiki/Inertia

Translation, Rotation, Rolling

- **Translational** motion: all particles in the object have the same instantaneous velocity (linear motion)
- **Rotational** motion: all particles in the object have the same instantaneous angular velocity
- **Rolling** motion: combination of translation and rotation

Notes

- In solving rotational motion problems you must chose a **rotational axis**.
- The object may return to its original angular position.

Circular Motion

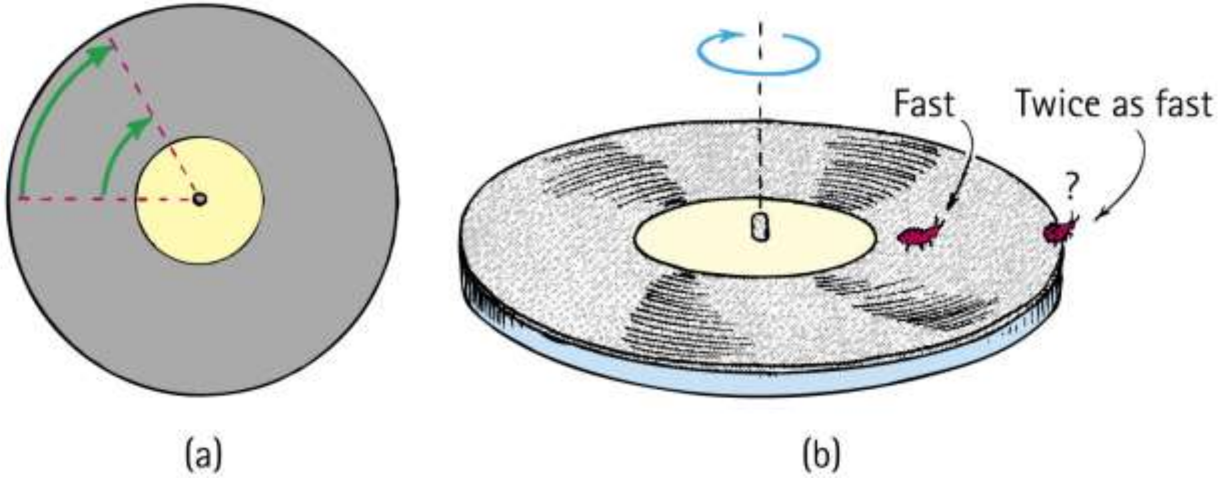
- When an object turns about an internal axis, it is undergoing circular motion or rotation.

- Circular Motion is characterized by two kinds of speeds:
  - tangential (or linear) speed.
  - rotational (or circular) speed.

Circular Motion—Tangential Speed

The distance traveled by a point on the rotating object divided by the time taken to travel that distance is called its *tangential* speed (symbol *v*).

- Points closer to the circumference have a higher tangential speed that points closer to the center.



Circular Motion – Rotational Speed

- Rotational (angular) speed is the *number of rotations or revolutions per unit of time* (symbol ω).
- All parts of a rigid merry-go-round or turntable turn about the axis of rotation in the same amount of time.
- So, all parts have the same rotational speed.

$$\text{Tangential speed} = \text{Radial Distance} \times \text{Rotational Speed}$$

$$v = r\omega$$

Angular Velocity

- When a wheel is <u>rotating uniformly about its axis, </u>the angle θ
- changes at a rate called ω, while the distance s changes at a rate called its velocity v.
- Then $s = r\,\theta$ gives
- $v = r\,\omega.$

Centripetal Force

- Any force directed toward a fixed center is called a *centripetal force*.
- *Centripetal* means "center-seeking" or "toward the center."

Example: To whirl a tin can at the end of a string, you pull the string toward the center and exert a centripetal force to keep the can moving in a circle.

Centripetal Force

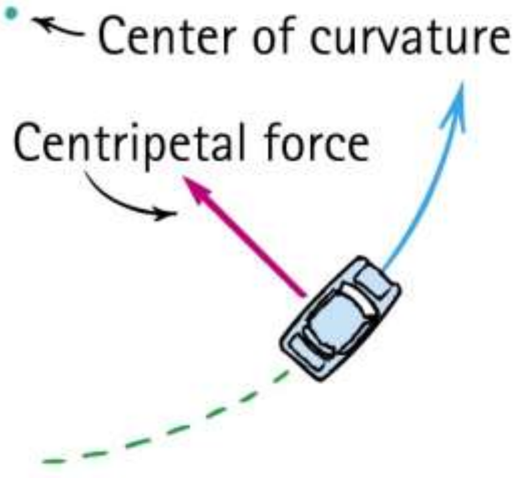
- Depends upon
  - mass of object.
  - tangential speed of the object.
  - radius of the circle.

- In equation form:

$$\text{Centripetal force} = \frac{\text{mass} \times \text{tangential speed}^2}{\text{radius}}$$

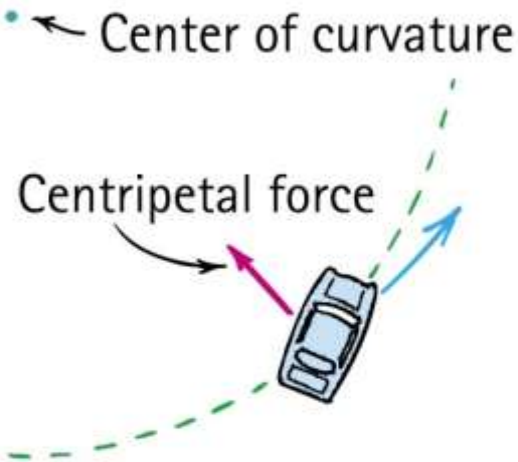
Centripetal Force—Example

- When a car rounds a curve, the centripetal force prevents it from skidding off the road.



(a)

- If the road is wet, or if the car is going too fast, the centripetal force is insufficient to prevent skidding off the road.



(b)

Centrifugal Force

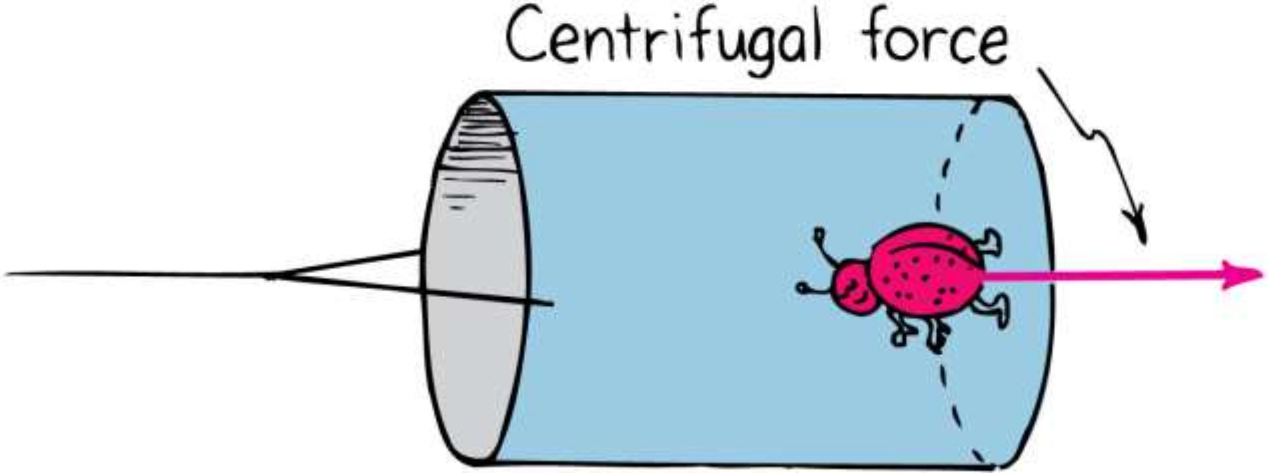
- Although centripetal force is center directed, an occupant inside a rotating system seems to experience an outward force. This apparent outward force is called ***centrifugal force.***

- *Centrifugal* means "center-fleeing" or "away from the center."

Rotating Reference Frames

- Centrifugal force *in a rotating reference frame* is a force in its own right – as real as any other force, e.g. gravity.

- Example:
  – The bug at the bottom of the can experiences a pull toward the bottom of the can.



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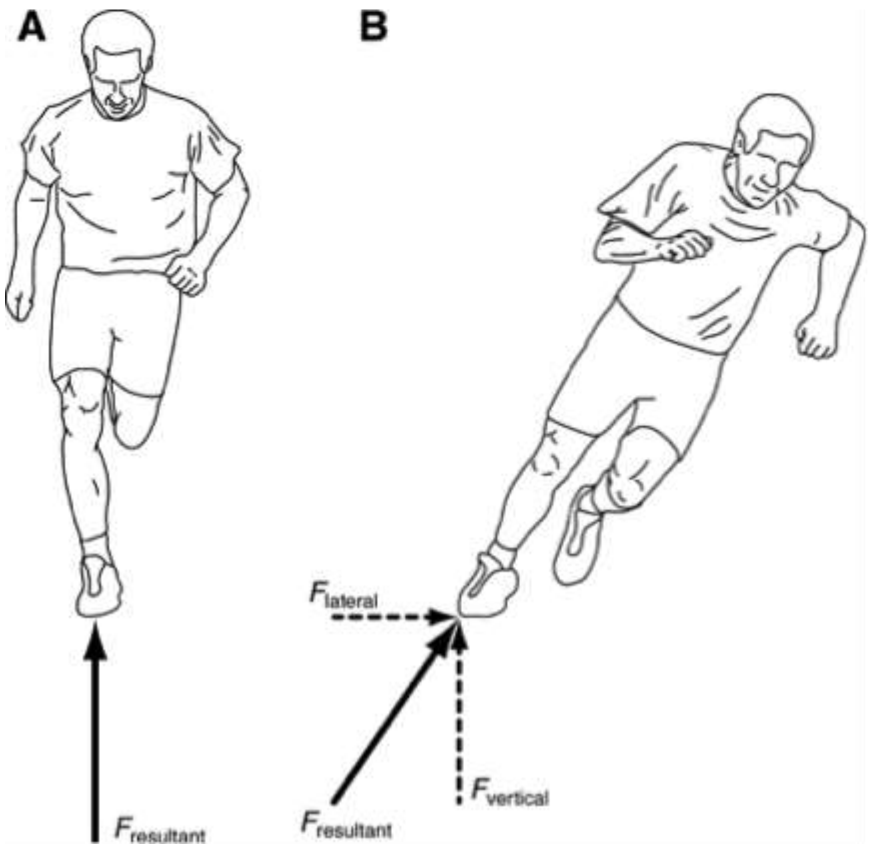
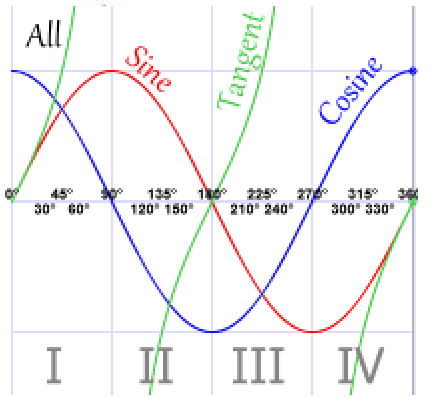
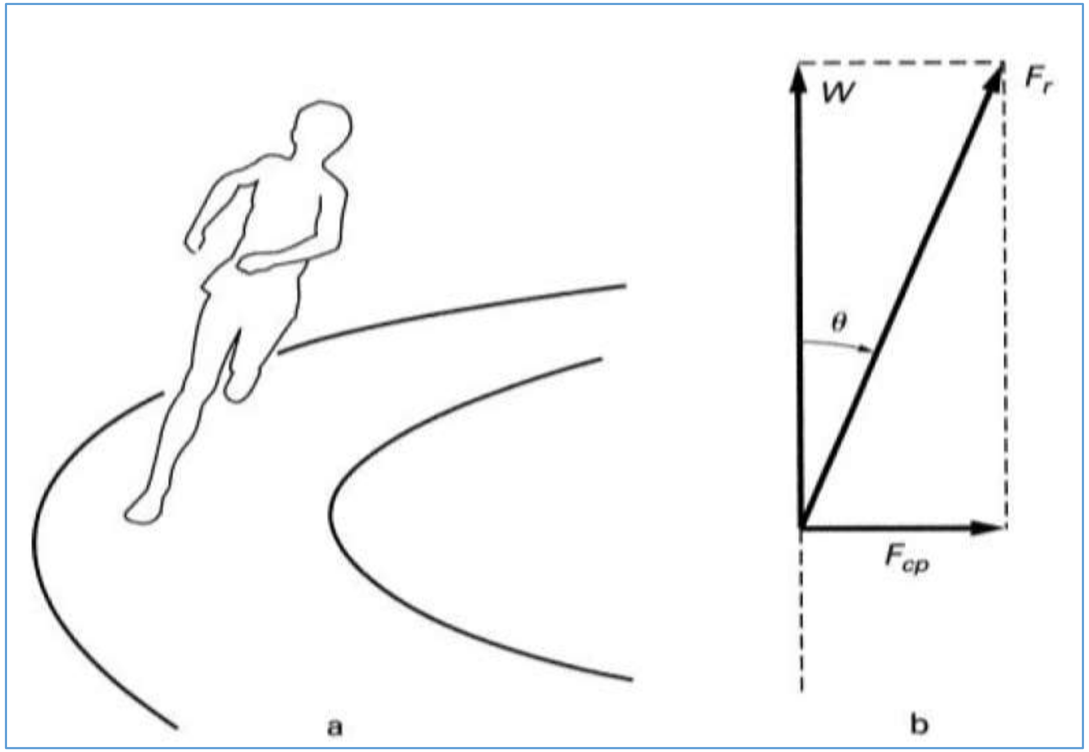


A Runner on a Curved Track



$$F_c = \frac{mv^2}{R} = \frac{Wv^2}{gR}$$



$$F_r \sin\theta = F_{cp} = \frac{Wv^2}{gR}$$

and

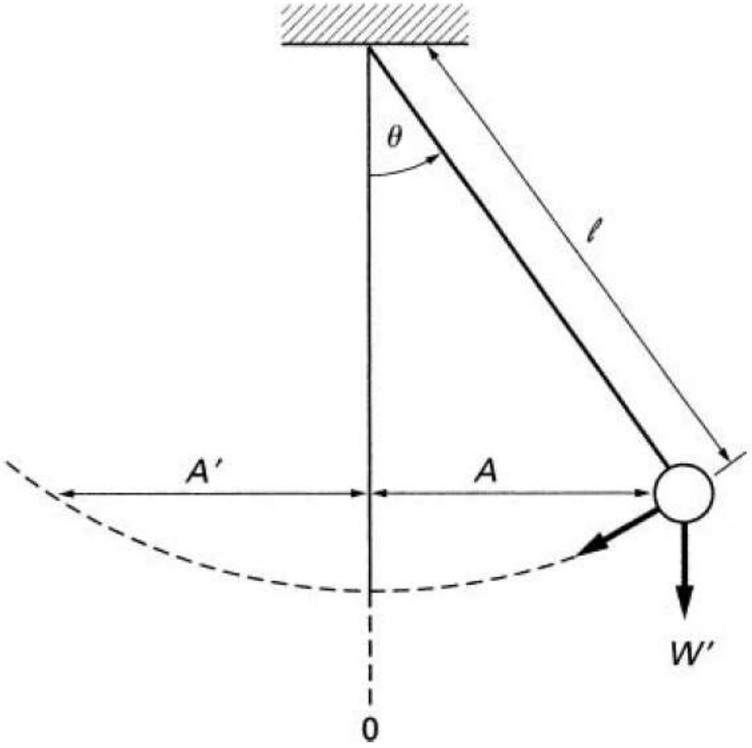
$$F_r \cos\theta = W$$

Therefore

$$\tan\theta = \frac{v^2}{gR}$$

(a) Runner on a curved track. (b) Forces acting on the foot of the runner.

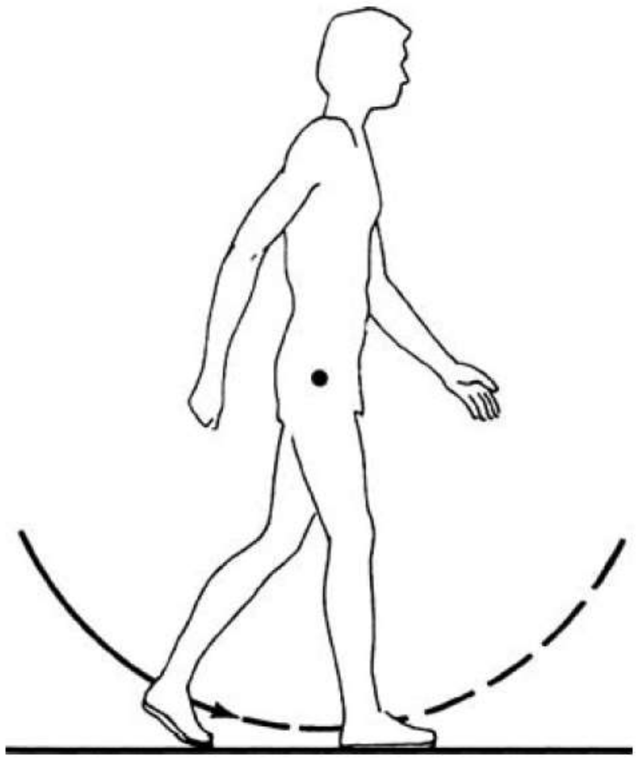
Pendulum of Life



The simple pendulum

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{\ell}{g}}$$

Although this expression for *T* is derived for a small-angle swing, it is a good approximation even for a relatively wide swing. For example, when the swing is through 120◦ (60◦ in each direction), the period is only 7% longer than predicted by Eq. above.

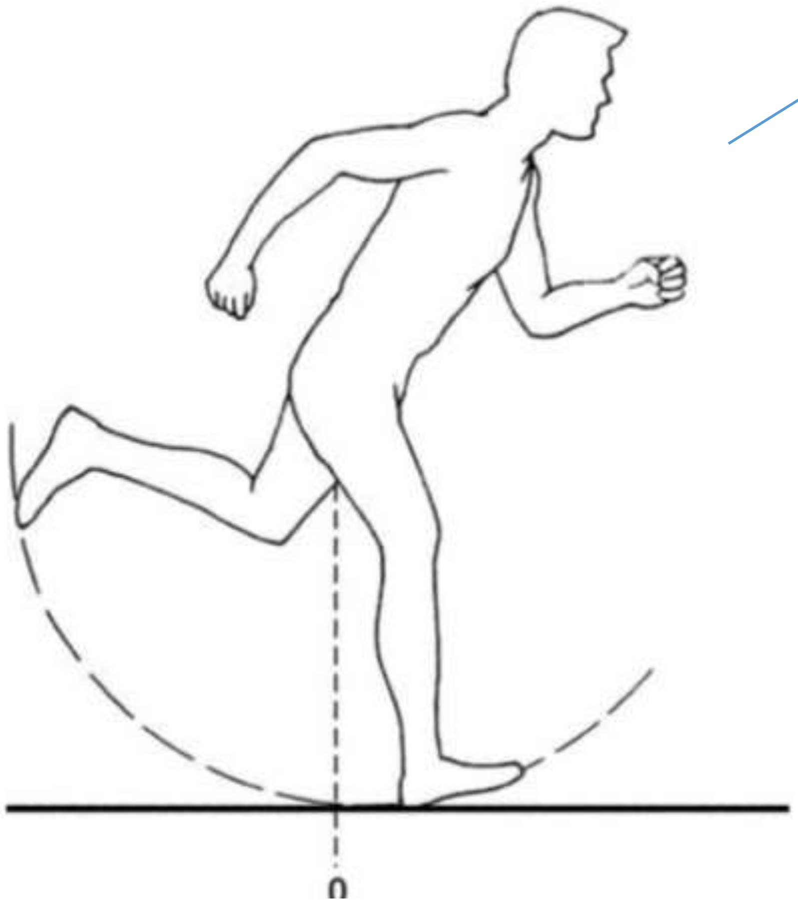


Walking

$$a_{\max} = \frac{4\pi^2 A}{T^2}$$

$$v_{\max} = \frac{2\pi A}{T}$$

Energy Expended in Running



$$E_r = \frac{1}{2} I \omega_{\max}^2$$

Fraction of Body Weight for Various Parts of the Body

	Fraction of body weight
Head and neck	0.07
Trunk	0.43
Upper arms	0.07
Forearms and hands	0.06
Thighs	0.23
Legs and feet	0.14
Total	1.00

From Cooper and Glassow [6-6], p. 174.

Angular Momentum

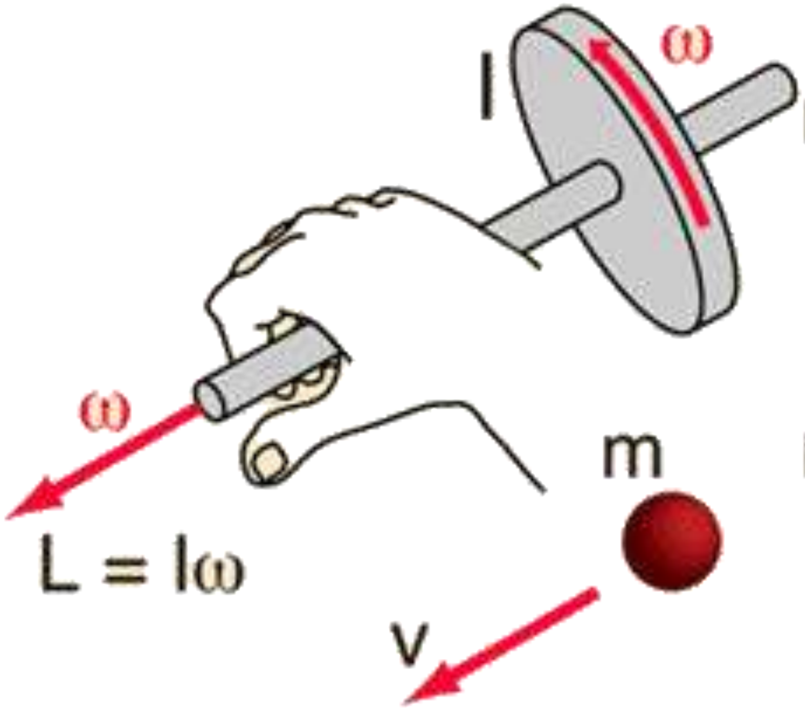
- The "inertia of rotation" of rotating objects is called *angular momentum.*
  - This is analogous to "inertia of motion", which was momentum.

- Angular momentum

$$= \text{rotational inertia} \times \text{angular velocity}$$

  - This is analogous to

$$\text{Linear momentum} = \text{mass} \times \text{velocity}$$



Angular Momentum = Moment of Inertia x Angular Velocity

$$L = I \times \omega$$

Linear Momentum = Mass x Velocity

$$p = m \times v$$

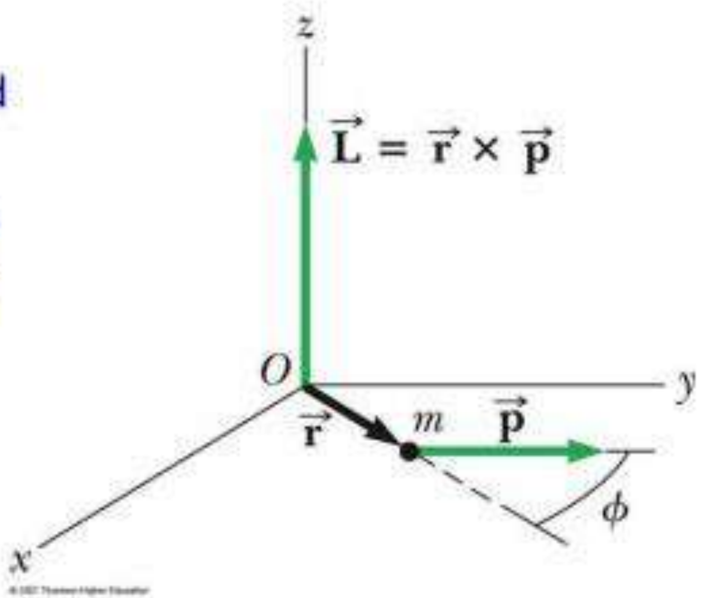
The x implies simple multiplication here.

Angular Momentum

- The instantaneous angular momentum $\vec{L}$ of a particle relative to the origin O is defined as the cross product of the particle's instantaneous position vector $\vec{r}$ and its instantaneous linear

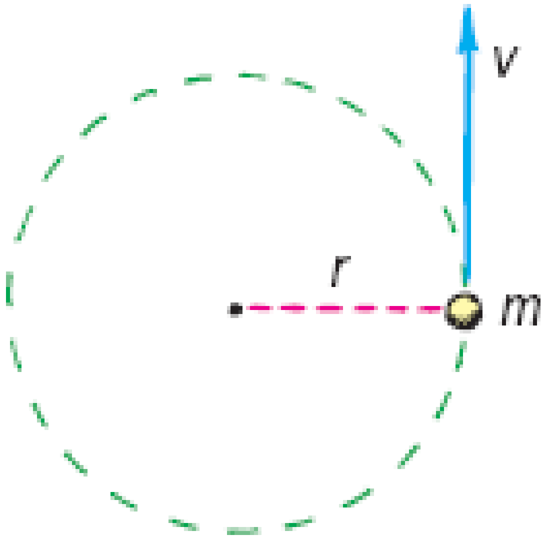
  momentum $\vec{p}$

- $$\vec{L} = \vec{r} \times \vec{p}$$



Angular Momentum

- Examples:
  - Whirling ball at the end of a long string
  - Planet going around the Sun



Angular Momentum

- An external net torque is required to change the angular momentum of an object.

- Rotational version of Newton’s first law:
  - **An object or system of objects will maintain its angular momentum unless acted upon by an external net torque.**

Conservation of Angular Momentum

The **law of conservation of angular momentum** states:

If **no external net torque** acts on a rotating system,

the **angular momentum of that system remains constant**.

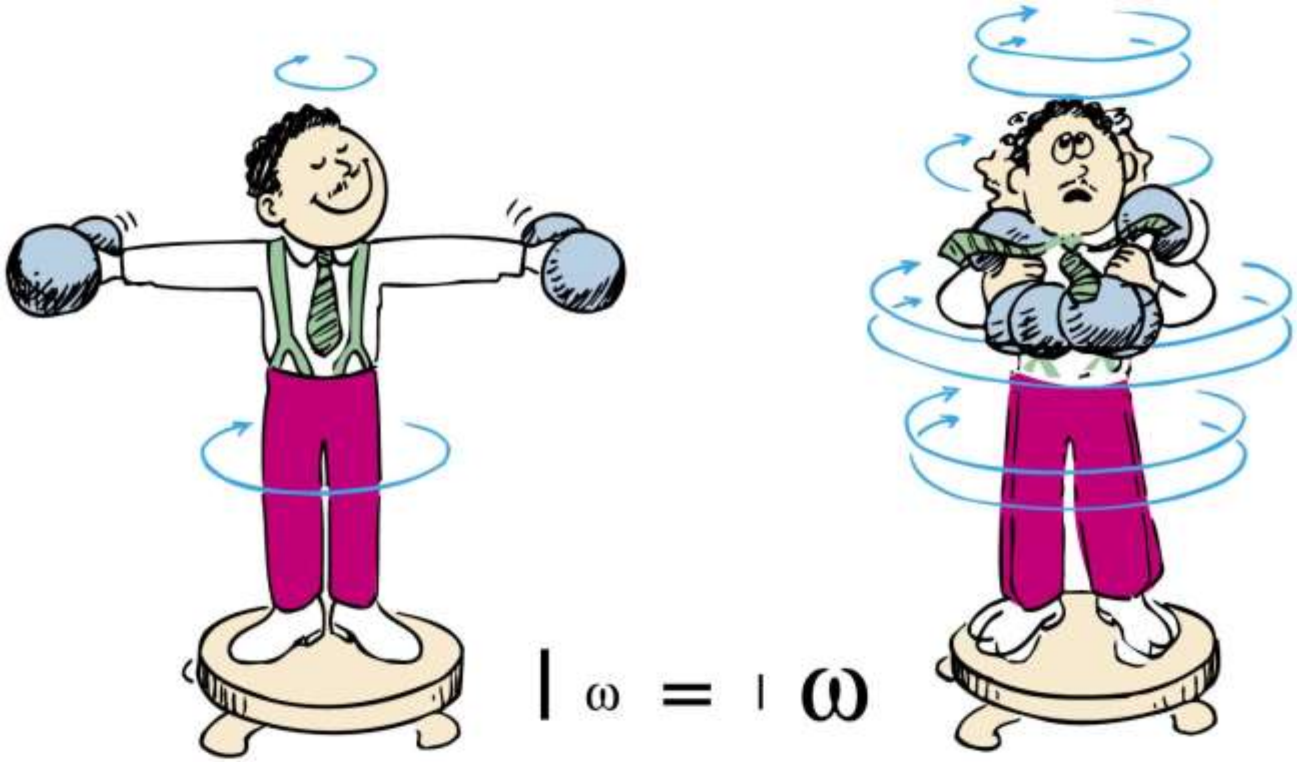
Analogous to the law of conservation of linear momentum:

If **no external force** acts on a system,

the total **linear momentum** of that system remains constant.

Example:

- When the man pulls the weights inward, his rotational speed increases!



Angular Momentum
CHECK YOUR NEIGHBOR

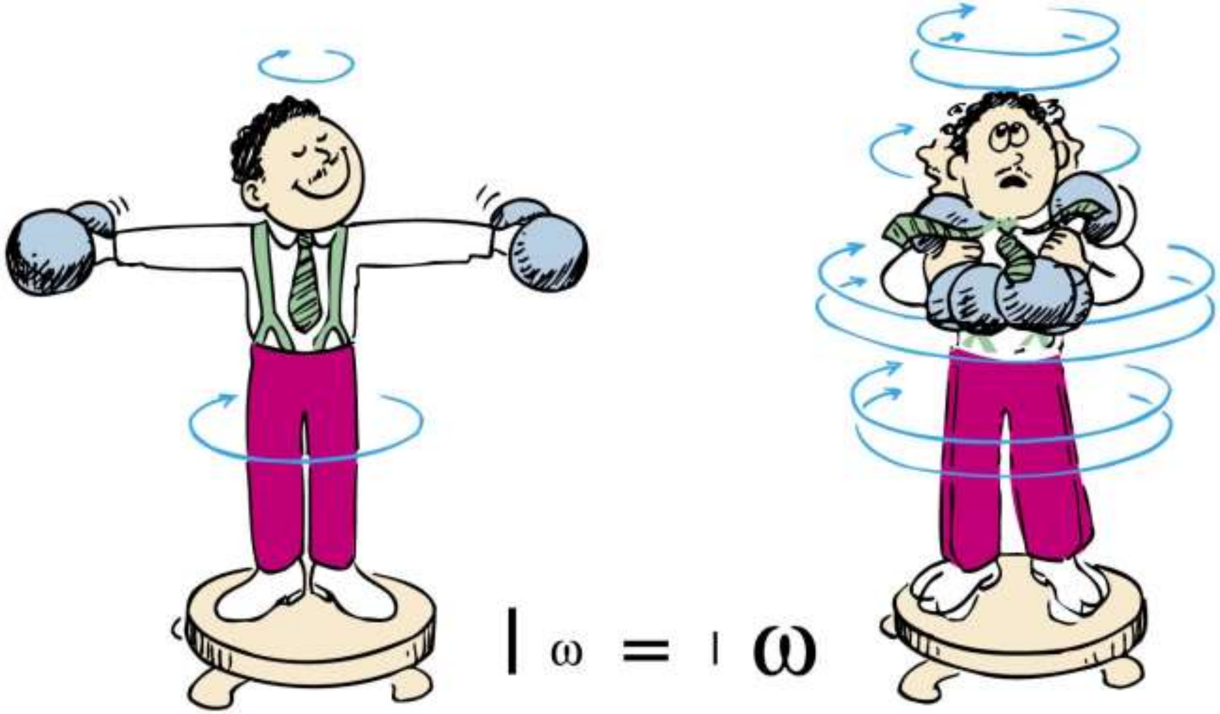
Suppose by pulling the weights inward, the rotational inertia of the man reduces to half its value. By what factor would his angular velocity change?

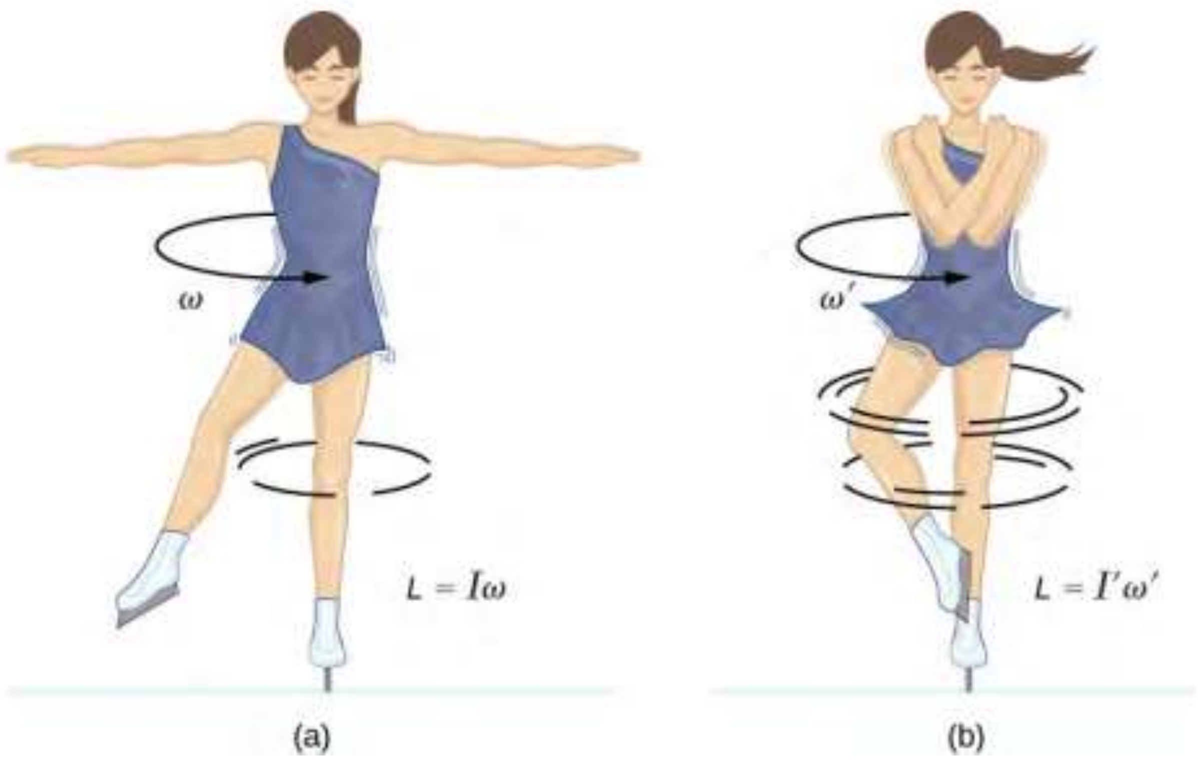
A. Double

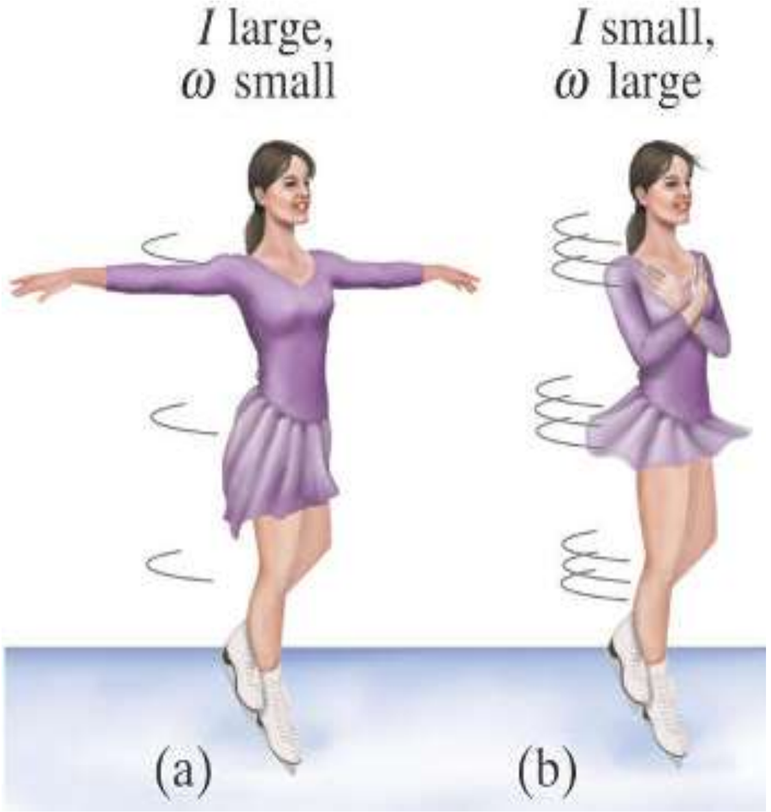
B. Three times

C. Half

D. One-quarter




ω
$L = I\omega$
(a)
ω'
$L = I'\omega'$
(b)


I large,
ω small
I small,
ω large
(a)
(b)

Angular Momentum
CHECK YOUR ANSWER

Suppose by pulling the weights in, if the rotational inertia of the man decreases to half of his initial rotational inertia, by what factor would his angular velocity change?

A. Double

B. Three times

C. Half

D. One-quarter

Explanation:

Angular momentum

$=$ rotational inertia $\times$ angular velocity

Angular momentum is proportional to "rotational inertia".

If you *halve* the rotational inertia, to keep the angular momentum constant, the angular velocity would **double**.

Angular Momentum and Moment of Inertia

- Let's recall the angular momentum
- $L = r\, m\, v = r\, m\, (\boldsymbol{\omega}\, r)$
- $L = m\, r^2\, \boldsymbol{\omega}$
- In a "rigid body", all parts rotate at the same angular velocity $\boldsymbol{\omega}$, so we can sum mr^2 over all parts of the body, to give
- $I = \Sigma\, mr^2$, the moment of inertia of the body.
- The total angular momentum is then
- $L = I\, \boldsymbol{\omega}$.

Conservation of Angular Momentum

- If there are no outside forces acting on a symmetrical rotating body, angular momentum is conserved, essentially by symmetry.
- The effect of a uniform gravitational field cancels out over the whole body, and angular momentum is still conserved.
- L also involves a direction, where the axis is the thumb if the motion is followed by the fingers of the right hand.

Applications of Conservation of Angular Momentum

- If the moment of inertial I_1 changes to I_2 , say by shortening r, then the angular velocity must also change to conserve angular momentum.
- $L = I_1 \boldsymbol{\omega}_1 = I_2 \boldsymbol{\omega}_2$
- Example: Rotating with weights out, pulling weights in shortens r, decreasing I and increasing $\boldsymbol{\omega}$.

Curving of spinning balls

Bernoulli's Equation (1738)

Magnus Force (1852)

Rayleigh Calculation (1877)

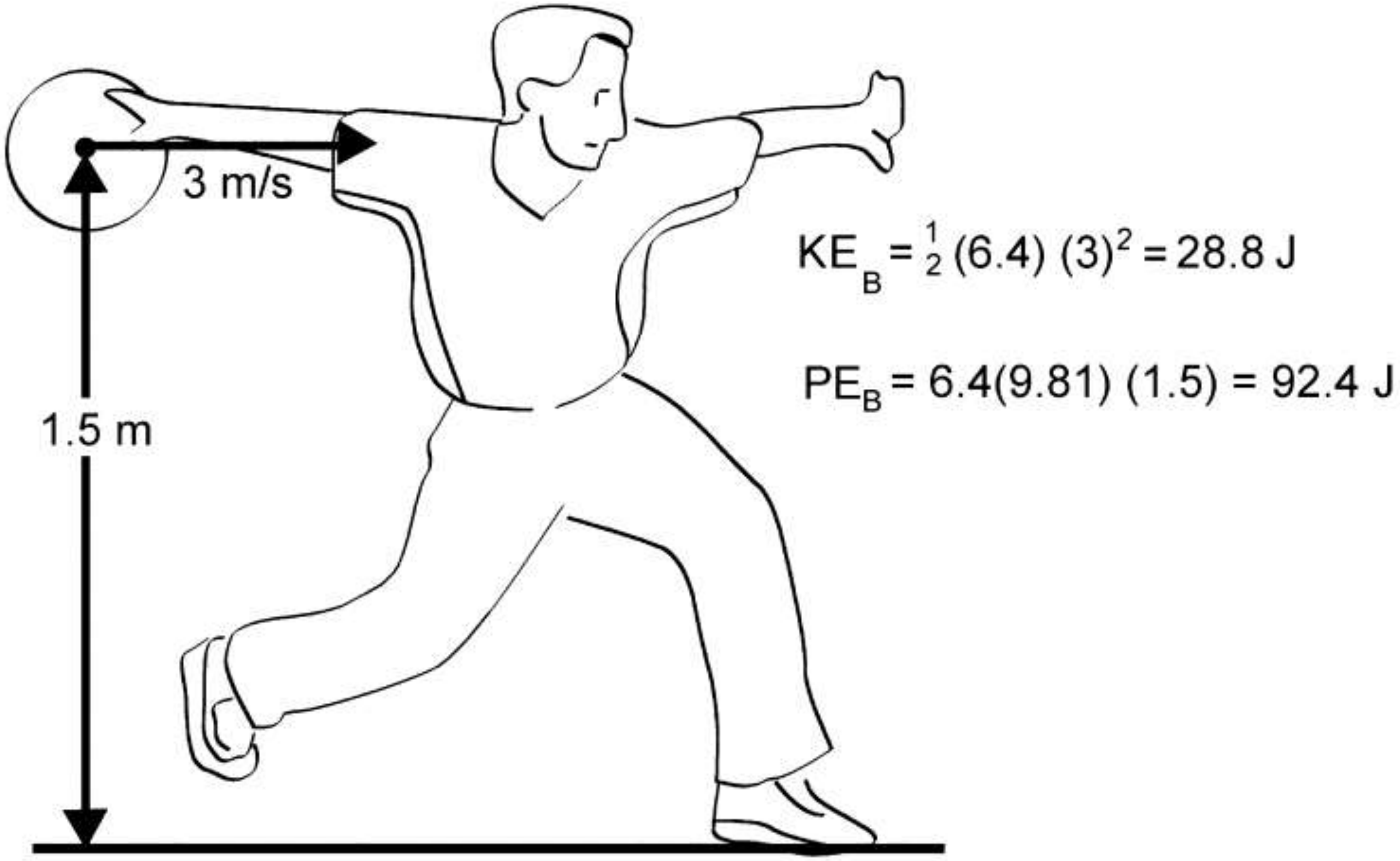
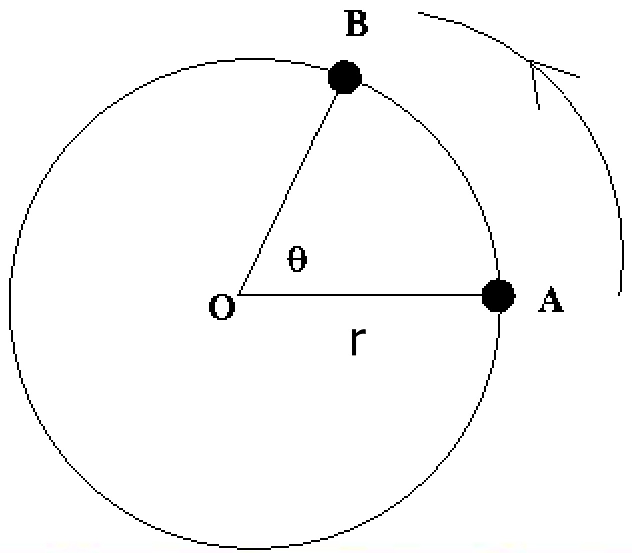


Figure 6.17. Raising a bowling ball in the approach stores more potential energy in the ball than the kinetic energy from the approach. The potential energy of the ball can be converted to kinetic energy in the downswing.

MATHEMATICS & PHYSICS BACKGROUNDS

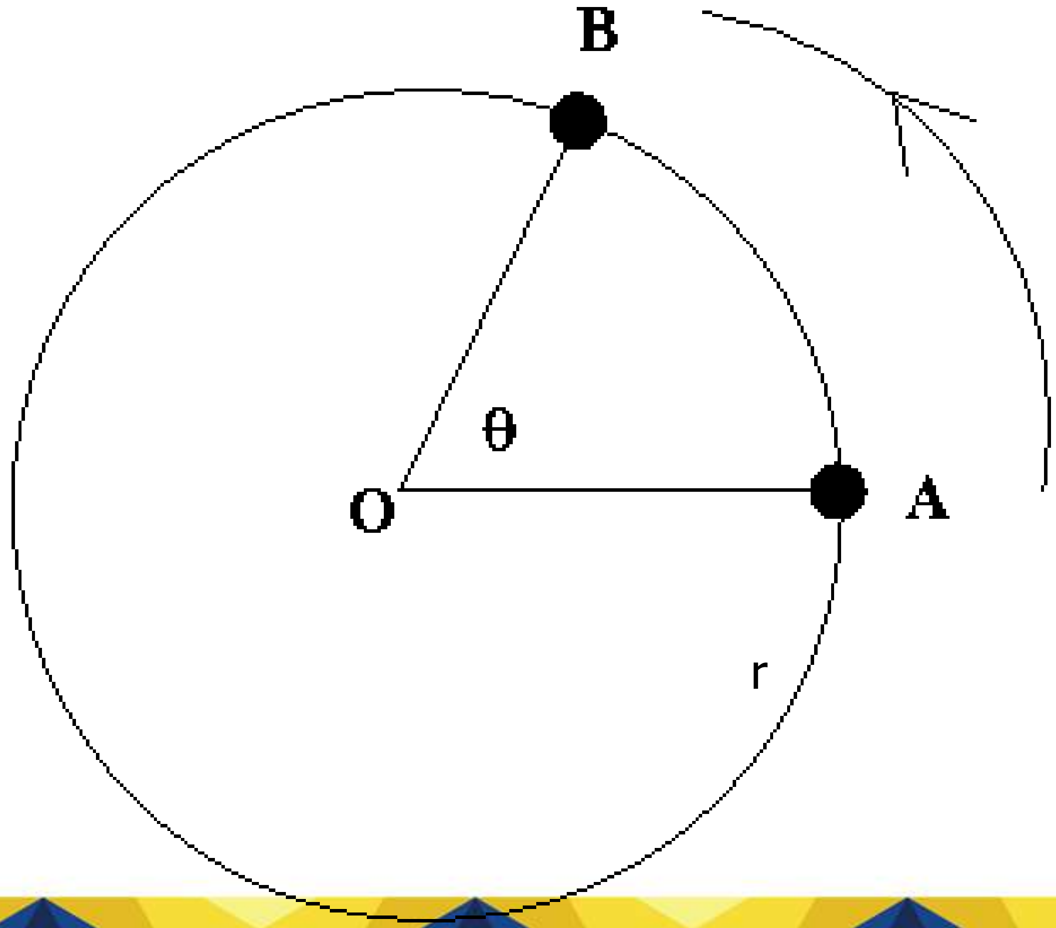
Polar Coordinates

- Radius r
- Angle θ measured counterclockwise from the + x axis



-Angular position, θ is positive counterclockwise from the + x axis

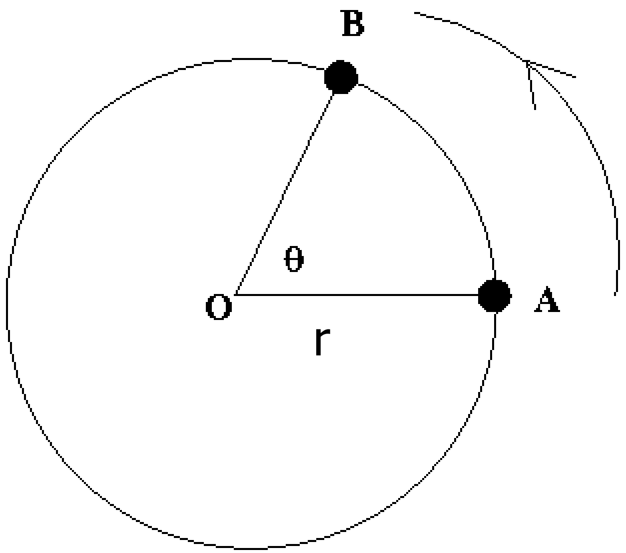
O is the point through which the axis of rotation passes.



Angular Displacement

$$\Delta\theta = \theta - \theta_o$$

(Final Angle-Initial Angle)



In this figure $q_o=0$

Units for Angular Displacement

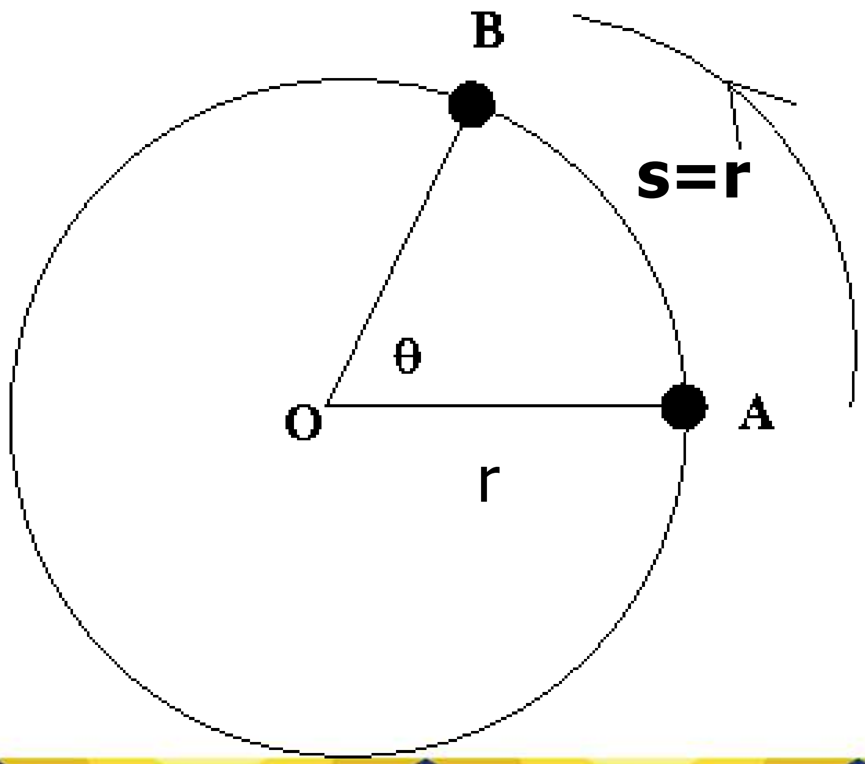
- Radian
  - One full revolution is 2π radians.
  - Radian is actually unitless.
  - The radian **is used** in the angular kinematics equations.

- Degree
  - 60 minutes in 1 degree, 60 seconds in 1 minute
  - The degree **is not used** in the angular kinematics equations.

Radian

- **Radian** (rad) is the angle subtended by an arc length, **s**, equal to the radius.

- When s=r, θ=1 rad



How many radii lengths fit in an arc length s?

$$\theta = \frac{s}{r} \rightarrow \boxed{s = \theta r}$$

θ in radians

r meters

s meters

- Why is the radian actually unitless?

$$1 \text{ rad} = \frac{360°}{2\pi} = 57.3°$$

$$\theta \text{ [rad]} = \frac{\pi}{180°}\theta \quad \text{[degrees]}$$

Average Angular Speed

$$\bar{\omega} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$$

SI unit : rad/sec

or 1/sec or sec^{-1}

- Note: common unit rpm (revolutions per minute)
- Angular speed will be positive if θ is increasing (counterclockwise)

Instantaneous Angular Speed

$$\omega \equiv \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Average Angular Acceleration

$$\bar{\alpha} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$$

Instantaneous Angular Acceleration

$$\alpha \equiv \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

- Units: rad/s² or s^{-2}

- Angular acceleration is positive if an object rotating counterclockwise is speeding up or if an object rotating clockwise is slowing down.

Area of Triangle

- given 2 sides and 1 angle

$$A = \tfrac{1}{2} bc \sin A = \tfrac{1}{2} ab \sin C = \tfrac{1}{2} ca \sin B$$

- given 3 sides

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

where $$s = \frac{a+b+c}{2}$$

Rotational Kinematic Equations

$$\omega_f = \omega_i + \alpha t$$

$$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$$

$$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$$

$$\theta_f = \theta_i + \frac{1}{2}\left(\omega_i + \omega_f\right)t$$

Table 10.1

Kinematic Equations for Rotational and Linear Motion Under Constant Acceleration

Rotational Motion About Fixed Axis	Linear Motion
$\omega_f = \omega_i + \alpha t$	$v_f = v_i + at$
$\theta_f = \theta_i + \omega_i t + \frac{1}{2}\alpha t^2$	$x_f = x_i + v_i t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_i^2 + 2\alpha(\theta_f - \theta_i)$	$v_f^2 = v_i^2 + 2a(x_f - x_i)$
$\theta_f = \theta_i + \frac{1}{2}(\omega_i + \omega_f)t$	$x_f = x_i + \frac{1}{2}(v_i + v_f)t$

Radius of gyration

- **FYI: radius of gyration is**

$$R_0 = \sqrt{\frac{I}{M}}$$

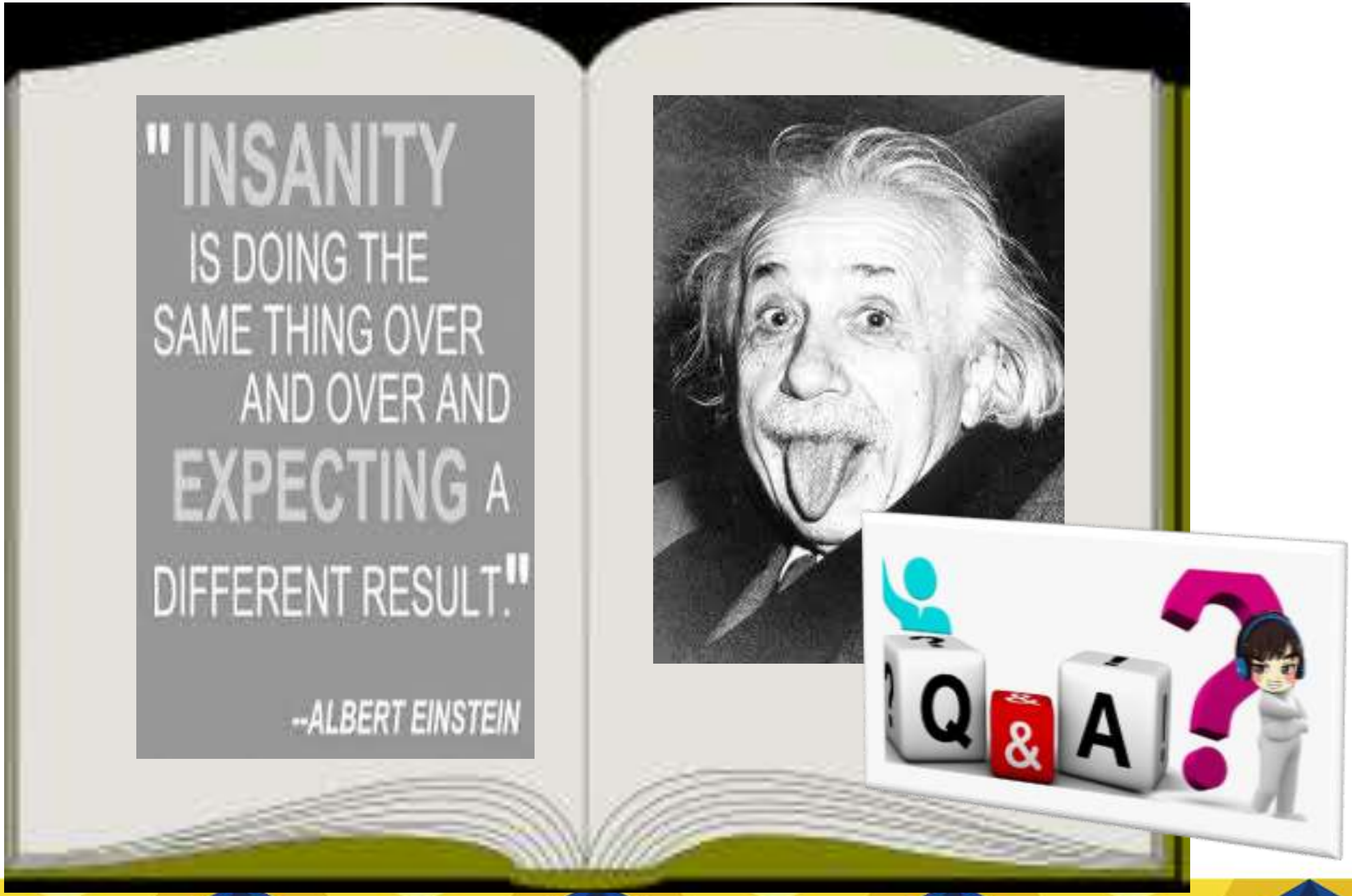
$$I = MR_0{}^2$$

$$T_R = \frac{I\omega^2}{2} = \frac{M(R_0\omega)^2}{2}$$

Tabel Trigonometri Sudut Istimewa

Sudut	0°	30°	45°	60°	90°	120°	135°	150°	180°
Sin	0	$\frac{1}{2}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}\sqrt{3}$	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0
Cos	1	$\frac{1}{2}\sqrt{3}$	$\frac{1}{2}\sqrt{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{2}\sqrt{2}$	$-\frac{1}{2}\sqrt{3}$	-1
Tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	$-\sqrt{3}$	-1	$-1/\sqrt{3}$	0
Cosec	∞	2	$\sqrt{2}$	$\frac{2}{3}\sqrt{3}$	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	∞
Sec	1	$\frac{2}{3}\sqrt{3}$	$\sqrt{2}$	2	∞	-2	$-\sqrt{2}$	$-\frac{2}{3}\sqrt{3}$	-1
Cot	∞	$\sqrt{3}$	1	$1/\sqrt{3}$	0	$-1/\sqrt{3}$	-1	$-\sqrt{3}$	∞

Monica Devi X4-15


"INSANITY
IS DOING THE
SAME THING OVER
AND OVER AND
EXPECTING A
DIFFERENT RESULT."
--ALBERT EINSTEIN
Q & A