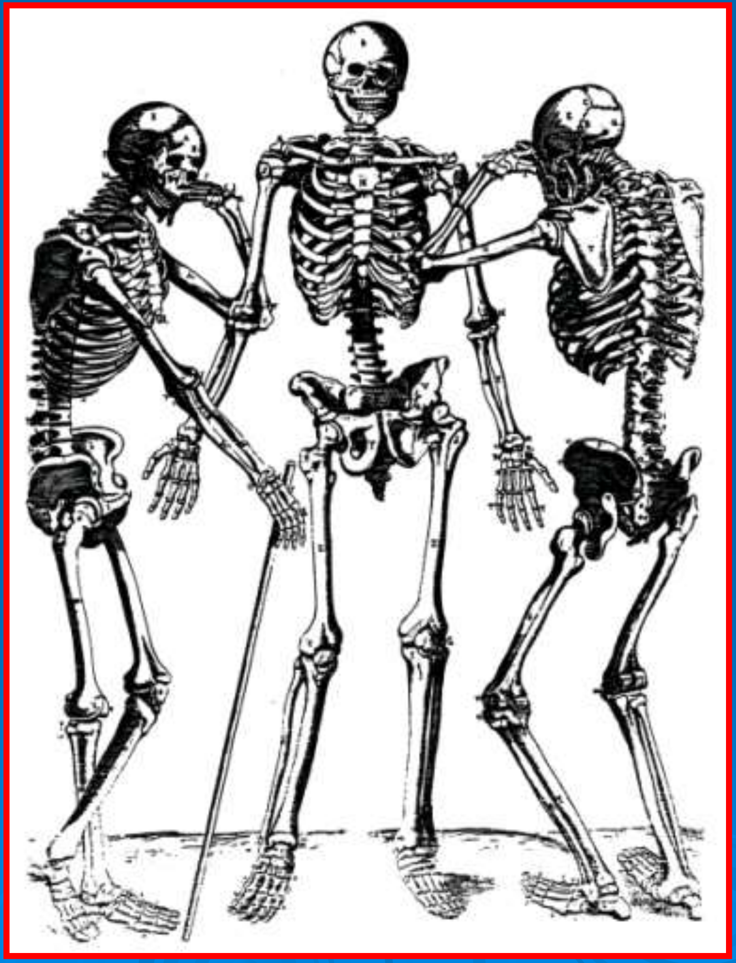
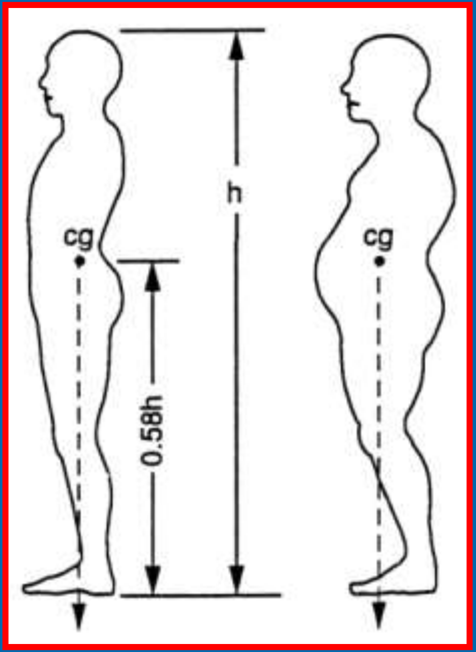
1. THE MECHANICS OF THE BODY

1.1 SKELETON, FORCES, AND BODY STABILITY



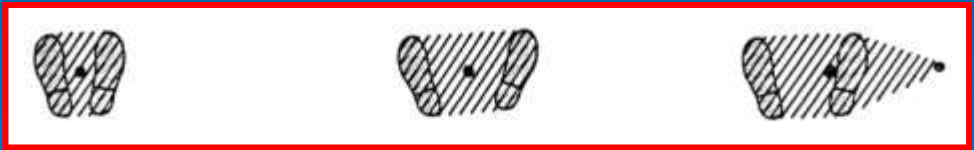
The main force acting on the body is the gravitational force!



(W= weight!) → $W = m \cdot g$

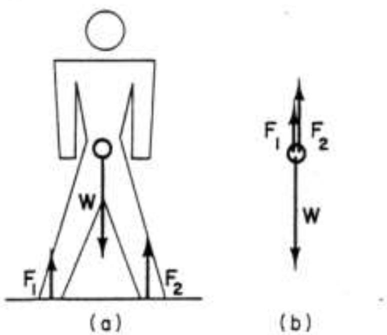
Stability of the body against the gravitational force is maintained by the bone structure of the skeleton!

Gravitational force W applies at the center of gravity **CG** of the body!



CG depends on body mass distribution! to maintain stability **CG** must be located between feet, if feet are far apart forces in horizontal direction F_x have to be considered

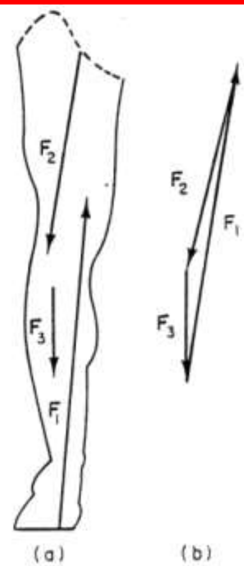
$$\sum \vec{F}_i = 0$$



$$\vec{F}_1 + \vec{F}_2 = \mathrm{W}$$

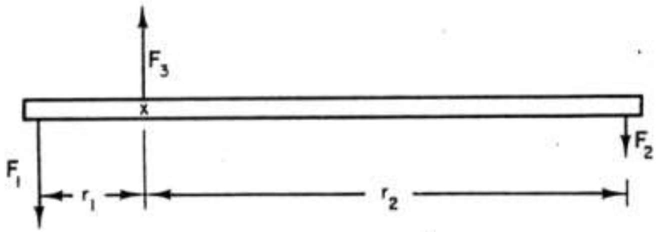
$\vec{F}_1$, $\vec{F}_2$ are the forces the ground applies to the feet

To maintain stability the vector sum of all forces applying at the **CG** must be zero!



in reality the forces applied to the leg are the vector sum of:
force of ground: $\vec{F}_1$
weight of body: $\vec{F}_2$
weight of leg: $\vec{F}_3$
in equilibrium: $\sum_i \vec{F}_i = 0$

The torque r causes a rotational movement around a pivot point!

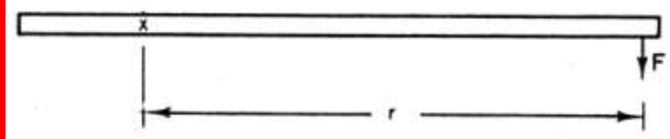


$$\sum_i \vec{\tau}_i = \sum_i \vec{r}_i \times \vec{F}_i = 0$$

Torque is defined by the force *F* applied at the distance *r* from the pivot point.

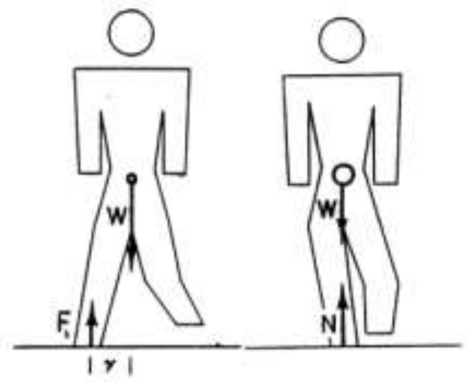
In rotational equilibrium (no rotation, constant rotation) to maintain stability for a person standing on one leg the torque requires to shift **CG** of body so, that:

$$\boxed{\Sigma_i \, \vec{\tau}_i = 0}$$



$$\vec{\tau} = \vec{r} \times \vec{F}$$

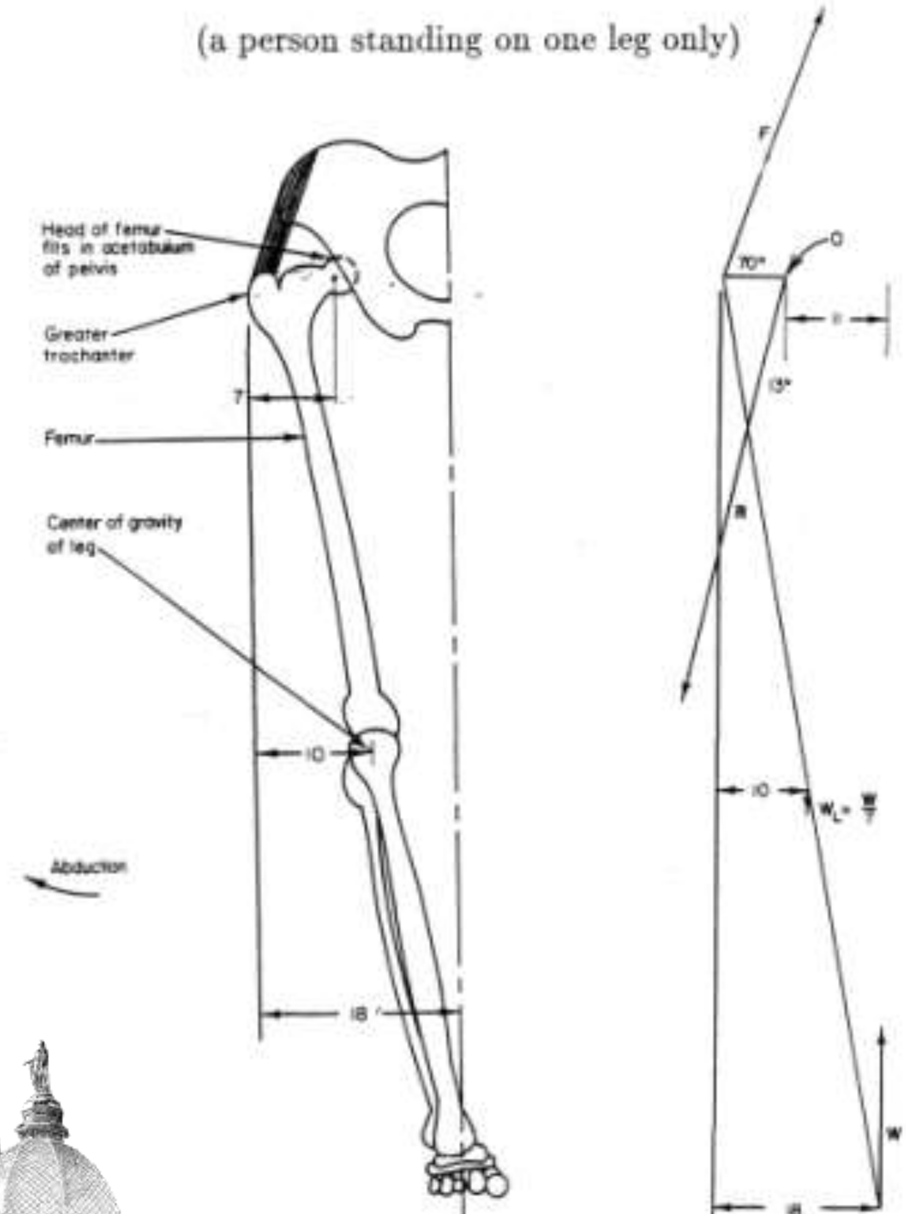
New CG: $\boxed{\vec{F}_1 = \vec{W}}$



torque for an average person of weight $\vec{W}$=80 kg·9.81 m/s^2 and $\vec{r}$=20cm, $\vec{\tau} \approx$16 N·m

EXAMPLE: FORCES ON THE HIP

(a person standing on one leg only)



$\vec{F}$: net force of the abductor muscles acting at the greater trochanter at $\approx 70^o$
$\vec{R}$: force at the acetabulum
$\vec{N}$: upward force of the ground on the bottom of the foot (in this case = $\vec{W}$)
$\vec{W}_L$: the weight of the leg, $\approx W/7$

vertical vector components: F_y

$$\sum F_Y = Fsin(70^o) - R_y - W/7 + W = 0$$

horizontal vector components: F_x

$$\sum F_x = Fcos(70^o) - R_x = 0$$

torque acting at the head of femur

$$\sum \tau = -Fsin(70^o)\cdot 7 - W/7\cdot(10-7) + W\cdot(18-7) = 0$$

this yields

$$11\cdot W - 3/7\cdot W - 6.6\cdot F = 0$$

FORCE IN THE ABDUCTOR MUSCLE: $F = 1.6W$

equations can be used to determine the force of the acetabulum

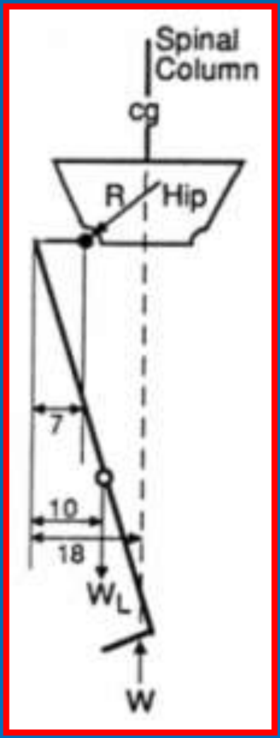
$$R_x = F\cos(70^o) = 1.6 \cdot W \cdot 0.342 = 0.55 \cdot W$$

$$R_y = F\sin(70^o) + 6/7 \cdot W = 1.6 \cdot W \cdot 0.94 + 0.86 \cdot W$$

FORCE ON THE ACETABULUM: $R = 2.4W$

for an 80 kg person: $F \approx 1260$ N; $R \approx 1880$ N; $W \approx 800$ N

EXAMPLE: FORCES ON THE HIP



a cane is needed to support body in the case of an affected hip
the cane is beneficial if used on the opposite side

assume abductor muscle is severed:

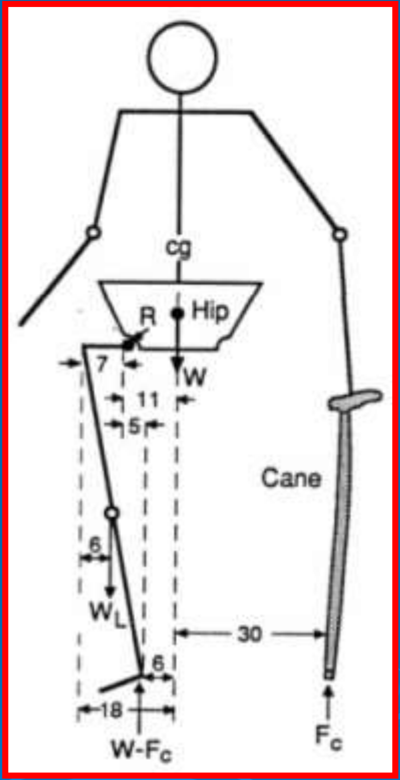
$$\Rightarrow \qquad \Sigma_i\, \tau_i = -\, 3/7 \cdot W + 11 \cdot W = 10.6 \cdot W$$

the torque at the head of the femur for an 80 kg person (one leg up):

$$\tau \approx 848 \text{ Nm}$$

with cane a percentage x of the body weigth is supported:

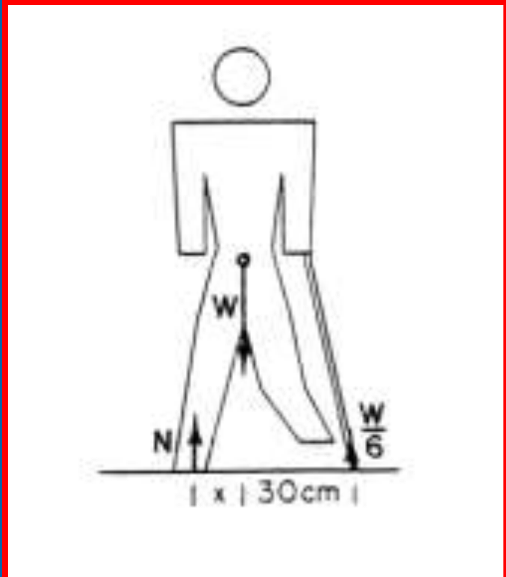
$$\Rightarrow \qquad \Sigma_i\, \tau_i = 10.6 \cdot (W - xW) - 30 \cdot xW$$
$$x \approx 1/4$$



(a person standing on one leg only)

a weak abductor muscle requires reduced force F:

additional support by cane
simplified assumption: force by ground up onto tip of the cane is vertical

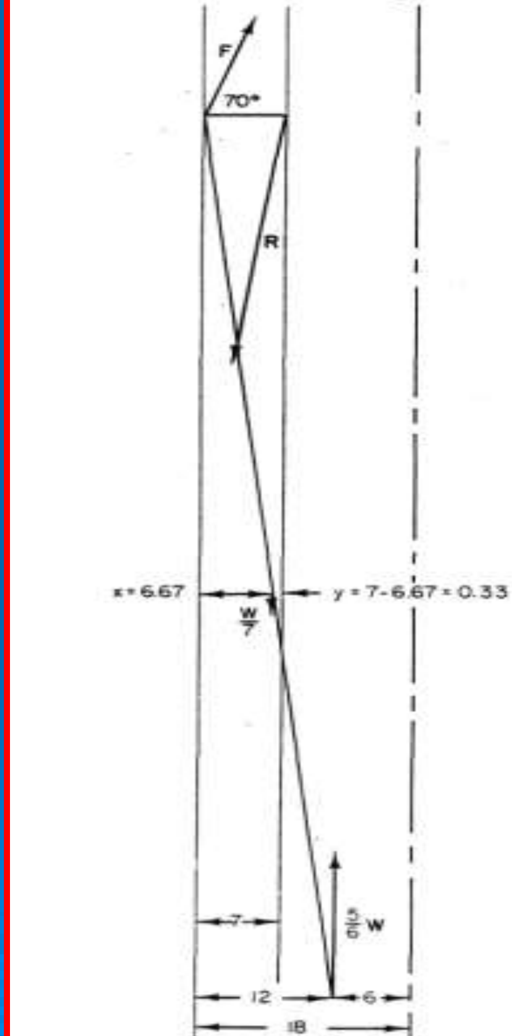


equilibrium condition:
$N + 1/6 \cdot W - W = 0$
$\Rightarrow \quad N = 5/6 \cdot W$

$\Sigma_i \tau_1 = 30 \cdot W/6 - x \cdot 5/6W = 0$

vector diagram for repositioned leg

$W_t = 1/6W$: force on tip
$r_t \approx 30$cm: distance tip-CG



to reduce force F, leg must be repositioned away from the midline (**CG**)

assumption: $\Rightarrow \qquad x \approx 6$ cm

this originates a torque at the head of the femur

$$\tau_F = N \cdot (18 - 7 - x) = 5/6 \cdot W \cdot 5 \text{ cm}$$

the weight of the leg $W_L \approx W/7$ has to be taken into account
it applies at the center of gravity of the leg
at 10/18 of horizontal distance between trochanter and foot

torque equation: $\qquad -F\sin(70^o) \cdot 7 - W/7 \cdot 0.33 + 5/6 W \cdot 5 = 0$

$$F = \mathbf{0.6}\ W$$

cane reduces force on abductor muscle to one-third of normal value
with cane supporting only one-sixth of weight

force of the acetabulum at head of femur:

$$F \cdot \cos(70^o) - R_x = 0$$

$$R_x = 0.2 \cdot W$$

$$F \cdot \sin(70^o) - R_y - W/7 + 5/6 \cdot W = 0$$

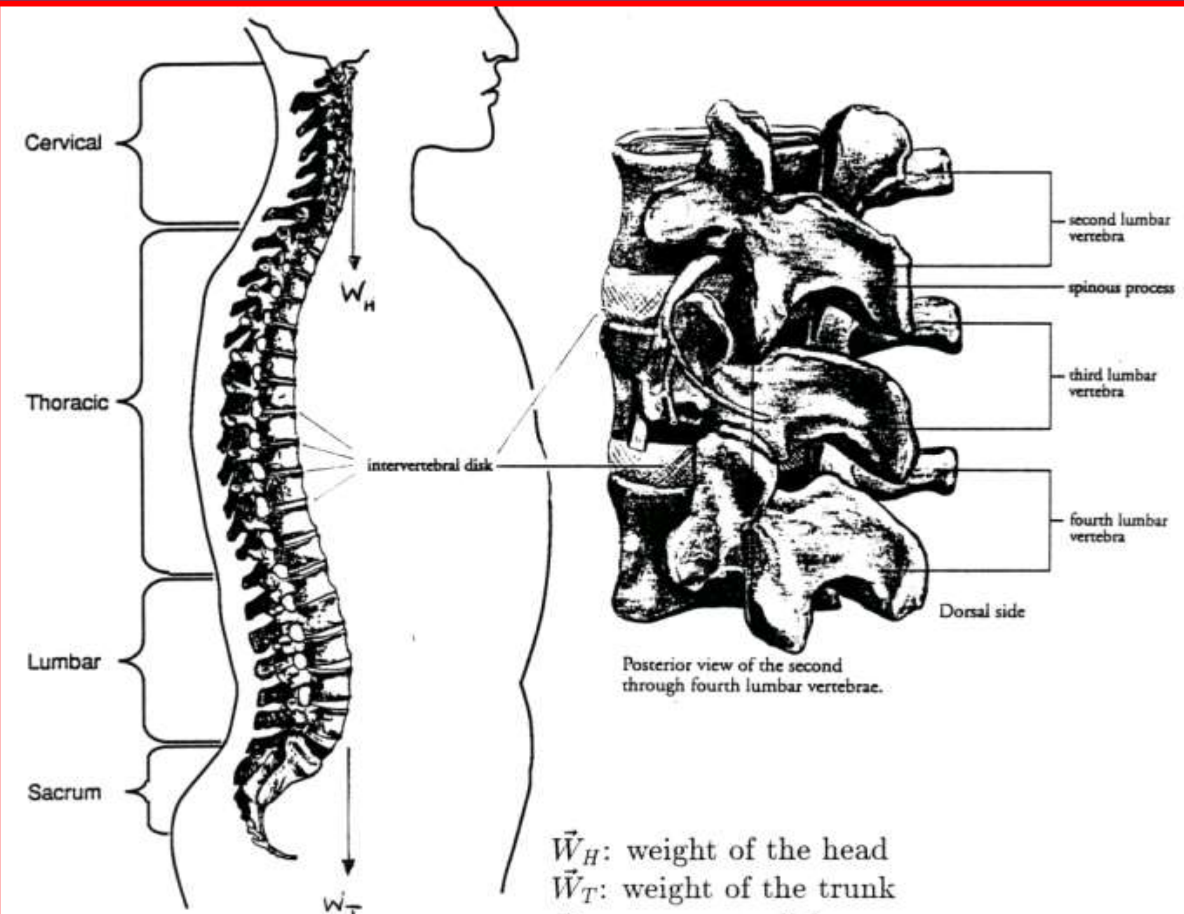
$$R_y = 1.25 \cdot W$$

resultant force: $\qquad R = \sqrt{R_x^2 + R_y^2} = 1.25 \cdot W$

cane reduces force of the acetabulum by one-half of normal value

EXAMPLE: FORCES ON THE SPINAL COLUMN

Spinal column supplies the main support for the head and trunk of the body. The spinal column is S-shaped to increase stability. Its bones, the vertebrae carry the load.
Vibrous disks between the vertebrae cushion the applied forces.



Posterior view of the second through fourth lumbar vertebrae.

$\vec{W}_H$: weight of the head
$\vec{W}_T$: weight of the trunk
P: pressure on disks
$P = W/A \approx$ constant

mass of the Head: $M_H \approx 3$ kg, $W_H \approx 30$ N
area of upper cervical vertebra: $A_{uC} \approx 5$ cm^2

$$\Rightarrow \qquad P \approx 6 \text{ N/cm}^2$$

pressure on the lower lumbar disk is about the same
taking into account the additional weight of the trunk
$W = W_H + W_T \approx 500$ N

$$A_{lL} = W/P \approx 80 \text{ cm}^2$$

$\Rightarrow$ cross sectional area of vertebra bones increases towards the lower sections of the spinal column to compensate for the increase in weight.

maximum pressure on disk before rupture: $P_{max} \approx 1100$ N/cm^2

cross sectional area of typical upper thoracic disk: $A \approx 10$ cm^2

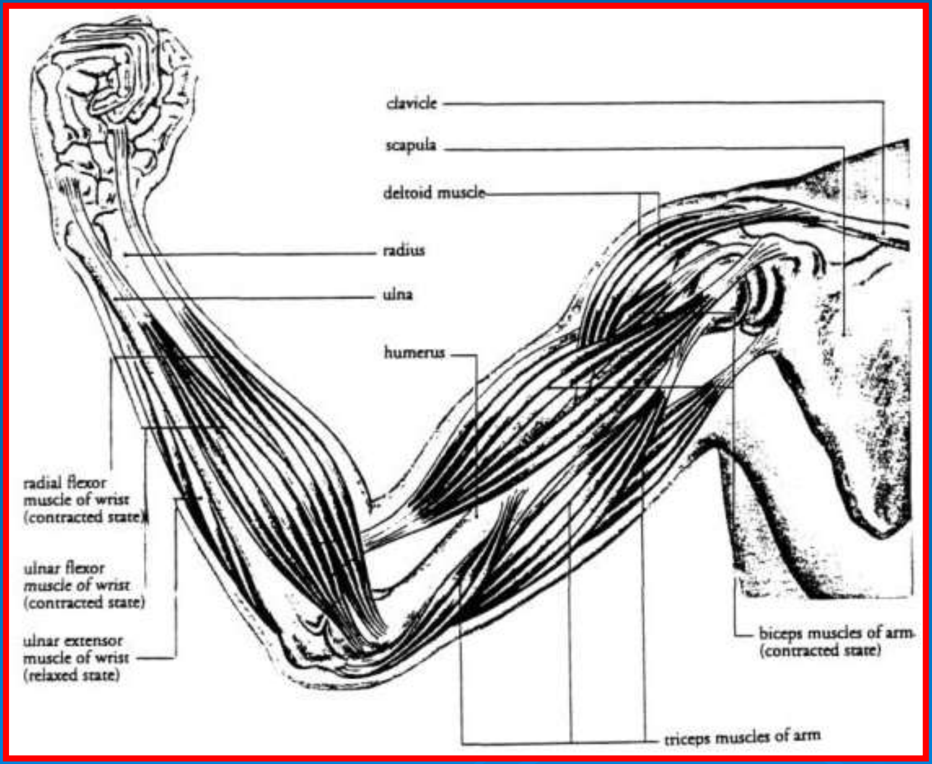
which force is necessary to rupture disk:

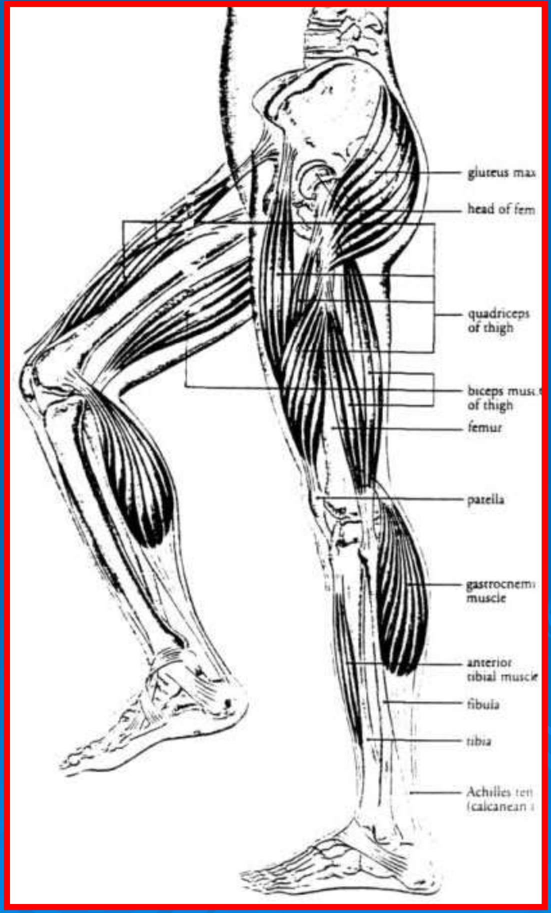
$$F_{max} \approx P_{max} \cdot A \approx 11{,}000 \text{ N}$$

(corresponds to a ton of weight)

1. THE MECHANICS OF THE BODY

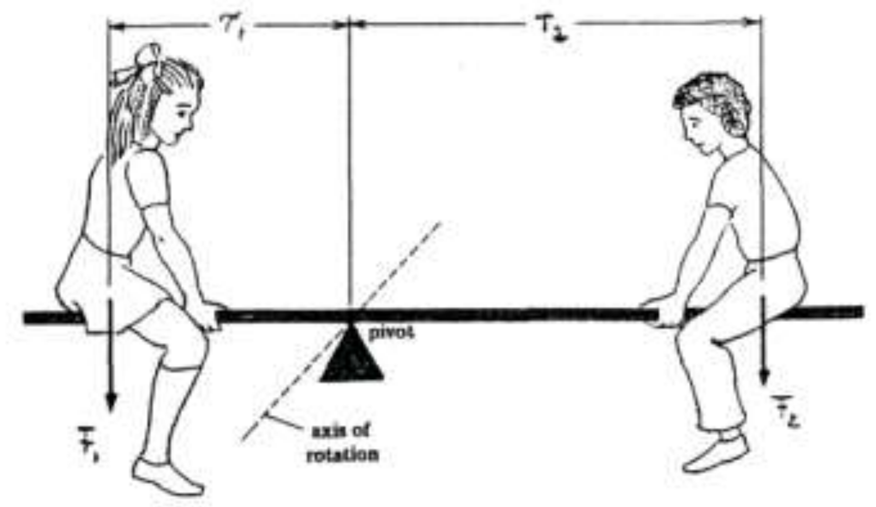
1.2 MUSCLES AND DYNAMICS OF BODY MOVMENT





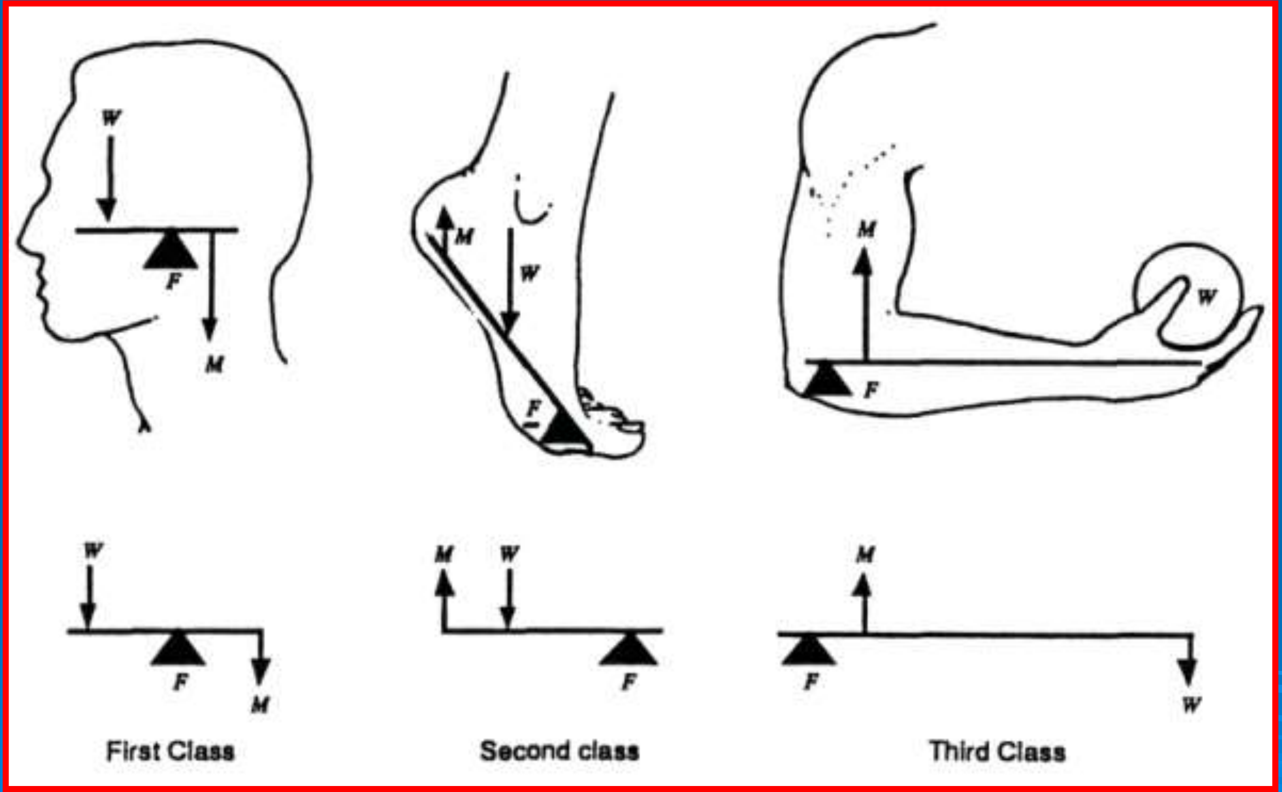
Body movements are controlled by muscle forces, initiated by contraction or extension of the muscles. Skeletal muscles control the movements of the body limbs.

Most of the muscle forces involve levers!



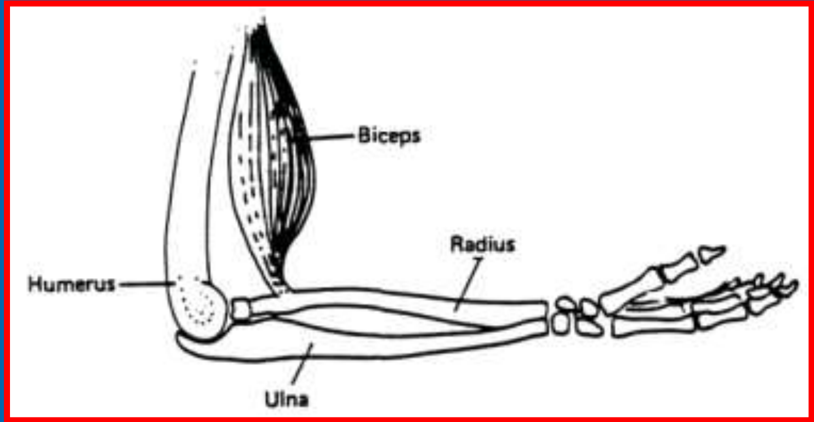
for the lever at rest: $\sum_i \tau_i = 0$

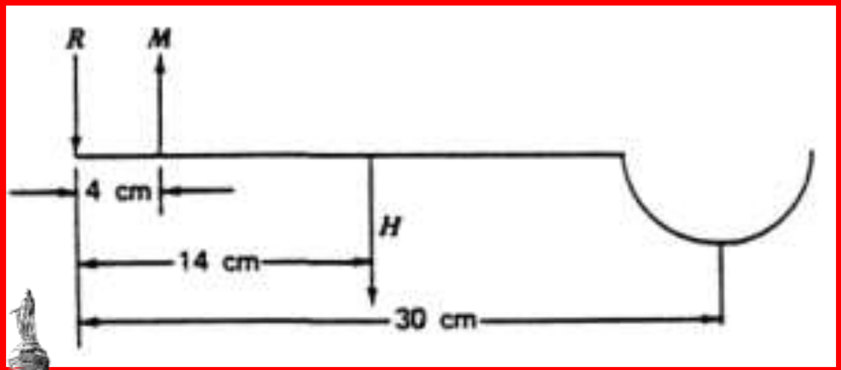
Three examples for lever systems, W is the applied weight, F is the force supporting the pivot point of the lever system, and M is the muscles force.



EXAMPLE: THE FOREARM AS LEVER SYSTEM

The biceps muscle pulls the arm upwards by muscle contraction with a force M the opposing force is the weight of the arm H at its center of gravity (CG)!





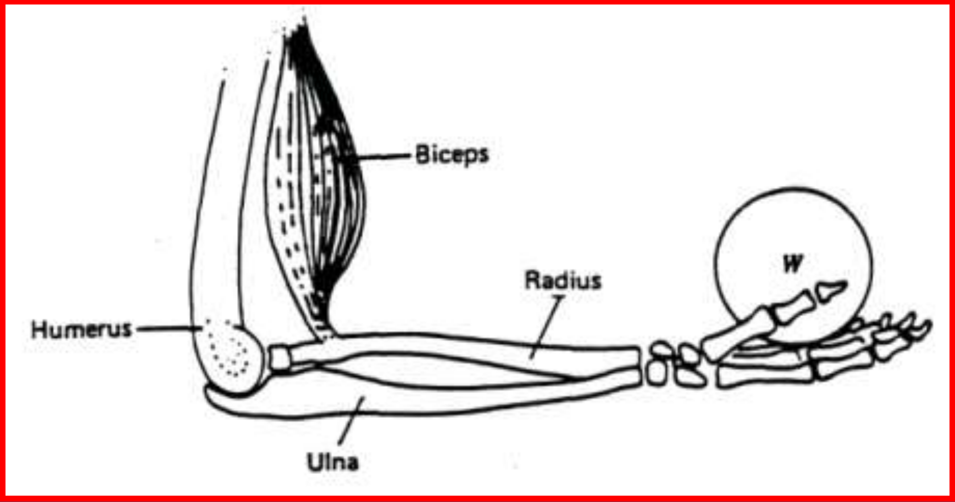
$$\sum_i \vec{\tau}_i = \vec{r}_1 \times \vec{M} + \vec{r}_2 \times \vec{H} = 0$$

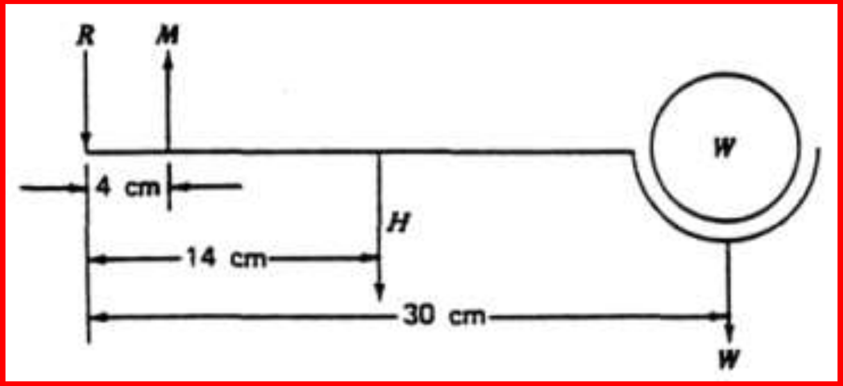
$$r_1 \cdot M - r_2 \cdot H = 4\mathrm{cm} \cdot M - 14\mathrm{cm} \cdot H = 0$$

(all forces apply perpendicular to the lever arm)
with $H \approx 15N$ (mass of the lower arm is approximately 3.3 lb)

$$M = 52.5\ \mathrm{N}$$

Biceps can be strengthened by weight W lifting this adds another force which has to be compensated by the muscle force.





$$\sum_i \vec{\tau}_i = \vec{r}_1 \times \vec{M} + \vec{r}_2 \times \vec{H} + \vec{r}_3 \times \vec{W} = 0$$

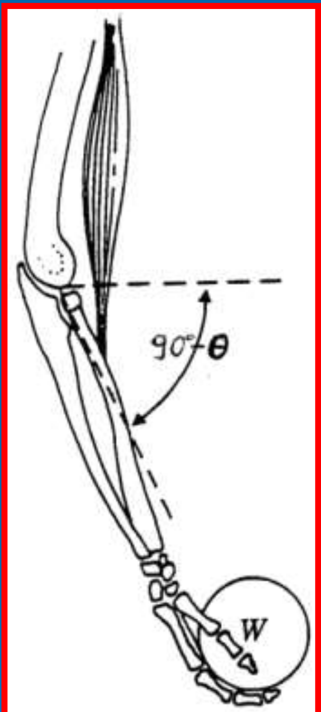
$$r_1 \cdot M - r_2 \cdot H - r_3 \cdot W = 4\text{cm} \cdot M - 14\text{cm} \cdot H - 30\text{cm} \cdot W = 0$$

$$M = (14/4 \cdot H + 30/4 \cdot W) = 52.5\ \text{N} + 7.5\ \text{W}$$

muscle force increases linearly with weight

for $W = 100\text{N}$ (22lb); $M = 802.5$ N

The lower arm can be hold by the biceps muscle at different angles θ. What muscle forces are required for the different arm positions?



$$\sum_i \vec{\tau}_i = \vec{r}_1 \times \vec{M} + \vec{r}_2 \times \vec{H} + \vec{r}_3 \times \vec{W} = 0$$

$$r_1 \cdot M \cdot sin\theta - r_2 \cdot H \cdot sin\theta - r_3 \cdot W \cdot sin\theta = 4cm \cdot M \cdot sin\theta - 14cm \cdot H \cdot sin\theta -$$

$$30cm \cdot W \cdot sin\theta = 0$$

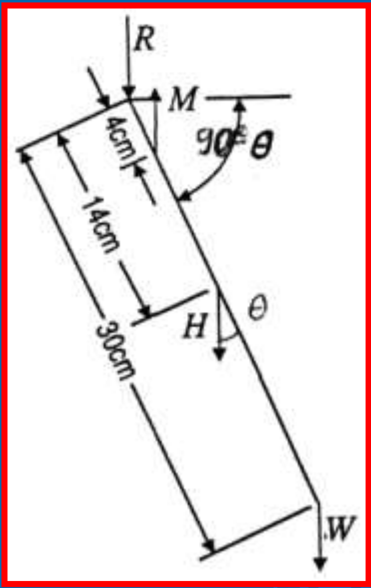
the sine-function cancels out

$$M = (14/4 \cdot H + 30/4 \cdot W) = 52.5\ N + 7.5\ W$$

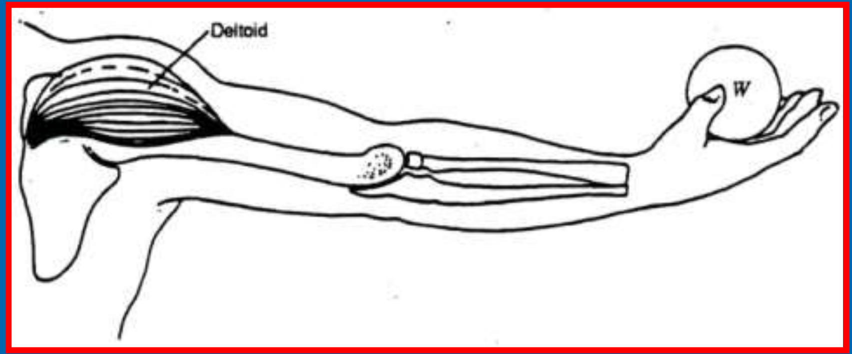
The applied muscle force is independent of the angle between the lower and upper arm

but muscle force still depends linearly on the weight

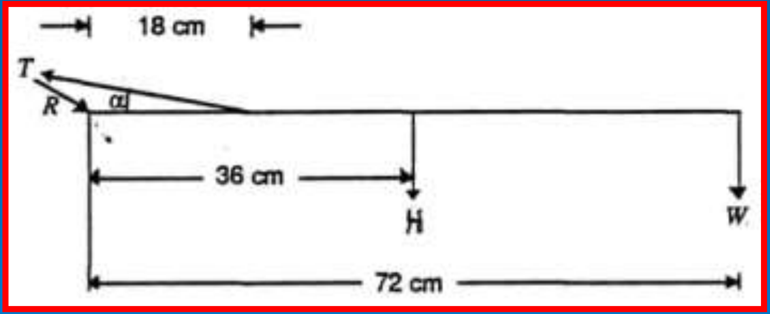
for $W = 100N$ (22lb); $M = 802.5\ N$



EXAMPLE: THE ARM AS LEVER SYSTEM



The deltoid muscle pulls the arm upwards by muscle contraction with a force T at a fixed angle *a* with respect to the arm the opposing force is the weight of the arm H at its center of gravity (CG) and the (possible) weight W hold in the hand!



$$\sum_i \vec{\tau}_i = \vec{r}_1 \times \vec{T} + \vec{r}_2 \times \vec{H} + \vec{r}_3 \times \vec{W} = 0$$

$$r_1 \cdot T \cdot \sin\alpha - r_2 \cdot H - r_3 \cdot W = 18\text{cm} \cdot T \cdot \sin\alpha - 36\text{cm} \cdot H - 72\text{cm} \cdot W = 0$$

$T = (14\text{cm} \cdot H + 30\text{cm} \cdot W)/4\text{cm} \cdot \sin\alpha$ with the mass of the arm $\approx$ 15 lb,

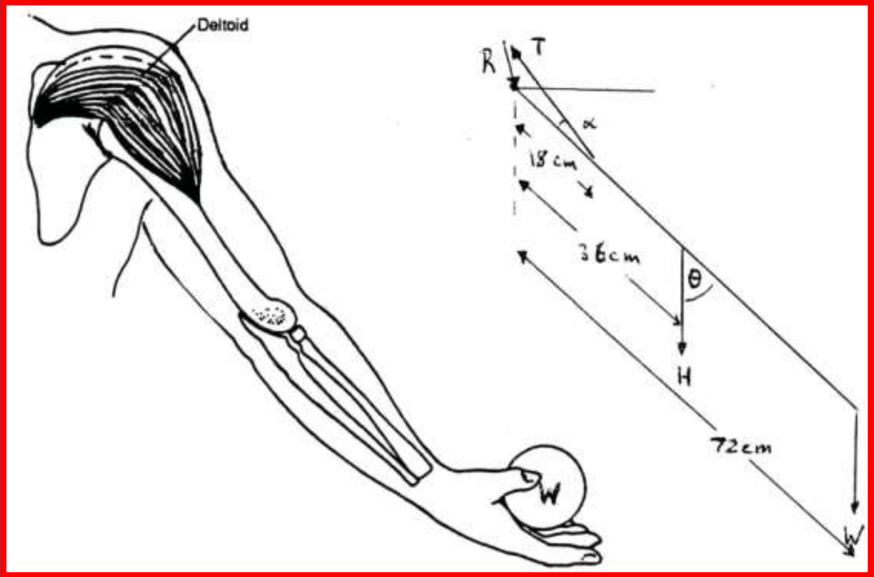
N$\approx$ 68 N and a weight of 45 N (10 lb)

$$\Rightarrow T = (36 \cdot 68\ \text{Ncm} + 72 \cdot 45\ \text{Ncm}) /\ 18\text{cm} \cdot \sin\alpha = 316\ \text{N}/\sin\alpha$$

muscle force depends on the angle of attachment $\alpha \approx 15^o$

$$T \approx 1220.9\ \text{N}$$

Consider arm at an angle θ hold by the deltoid muscle ($\alpha \approx 15°$)



$$\sum_i \vec{\tau}_i = \vec{r}_1 \times \vec{T} + \vec{r}_2 \times \vec{H} + \vec{r}_3 \times \vec{W} = 0$$

$r_1 \cdot T \cdot sin\alpha$ - $r_2 \cdot H \cdot sin\theta$ - $r_3 \cdot W \cdot sin\theta$ = 18cm·T·$sin\alpha$ - 36cm·H·$sin\theta$ - 72cm·W·$sin\theta$ = 0

T = (1/18cm)·(36cm·H +72cm·W)·($\sin\theta/\sin\alpha$) using the same force parameters as in the previous example

$\Rightarrow$ T = (1/18cm)·(36·68 Ncm + 72·45 Ncm)·($\sin\theta/\sin\alpha$) = 316 N·($\sin\theta/\sin\alpha$)

muscle force depends in this case also on the angle θ
Assuming an approximately constant angle of attachment $\alpha \approx 15^o$

$$T \approx 1220.9 \text{ N}\cdot\sin\theta$$

θ=60^o $\rightarrow T$ = 1057 N $\qquad$ θ=45^o $\rightarrow T$ = 863 N

θ=30^o $\rightarrow T$ = 610 N $\qquad$ θ=10^o $\rightarrow T$ = 212 N

1. THE MECHANICS OF THE BODY

1.3 PHYSICS OF BODY CRASHING

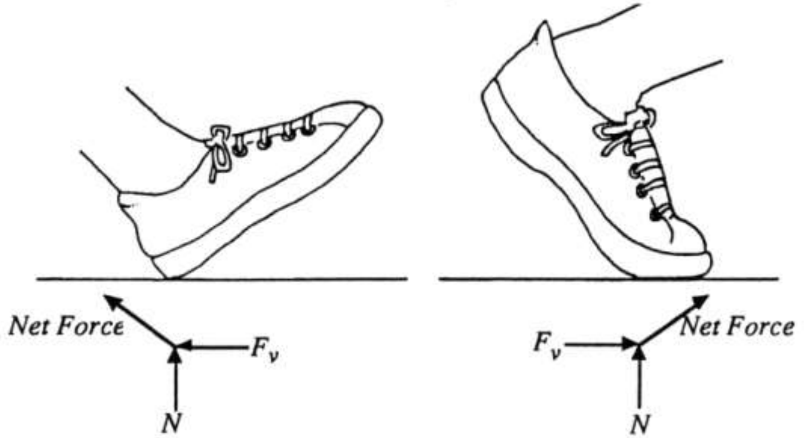
"It goes from zero to 60 in about 3 seconds."

If a body of mass *m* is in constant motion no acceleration or deceleration occurs !

$$\vec{v} = d\vec{x}/dt = \text{const}$$

Acceleration *a* can be caused by leg muscle force *F* !

$$\vec{a} = \vec{F}_m/m$$



Deceleration can be caused by friction, muscle force or external forces (by running into a wall for example).

Friction occurs between a moving surface and a surface at rest:

$$\vec{F}_f = \mu_k \vec{N}$$

N is the normal force!

μ_k is coefficient for kinetic friction:

for rubber-concrete: $\mu_k \approx 0.8$

joints between bones: $\mu_k \approx 0.003$

As smaller the coefficient as less resistance by frictional forces!

For a walker of $N \approx W \approx 800$ N (m=82kg):

Friction force: $F_f \approx 640$ N

deceleration: $a \approx 7.8$ m/s^2

Accelerating muscle forces maintain a constant walking speed!

When the body bumps into a solid object (like a wall) rapid deceleration *a* occurs:

$$\vec{a} = d\vec{v}/dt \approx \Delta\vec{v}/\Delta t$$

The decelerating force F_d applied by the wall to the body (or to whatever body part which hits first) causes pressure P_d which causes deformation:

$$\vec{F}_d = m \cdot \vec{a} \quad \vec{P}_d = \vec{F}_d / \mathrm{A}$$

A is the surface area of the body or body part exposed to the force.

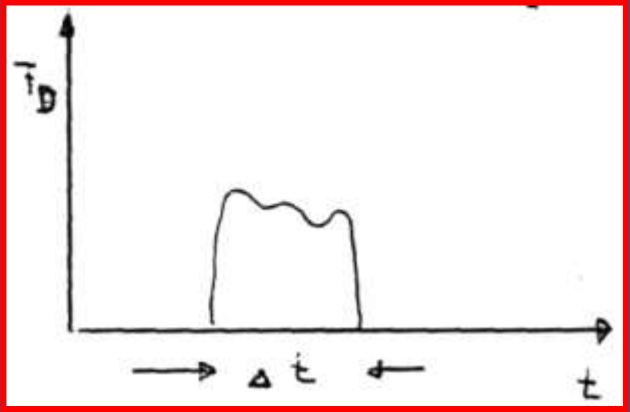
Force is only applied over the time period Δt until complete stop.

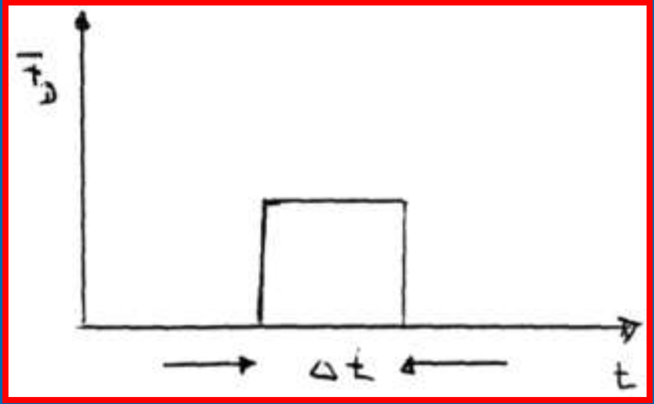
Therefore:

$$\Delta\vec{v} \approx \vec{v}_i$$

$\vec{v}_i$ is the velocity at impact

To calculate the impact force the time structure of the deceleration process needs to be known.





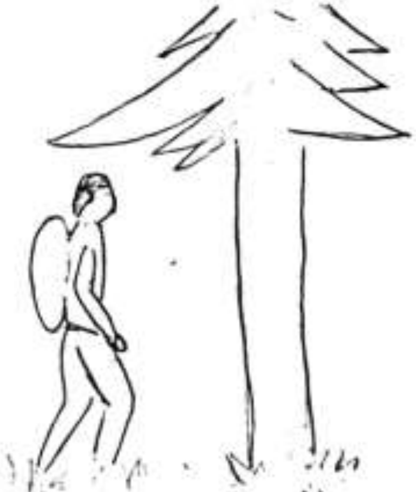
Approximation: treatment of force as a square pulse actual time structure may depend on particular impact

hard impact $\Rightarrow$ no deformation $\Rightarrow$ Δt is short

soft impact $\Rightarrow$ large deformation $\Rightarrow$ Δt is long

Increase of Δt reduces forces and therefore the danger of deformations causing fractures and other damages

EXAMPLE The unobservant hiker The hiker (m=82 kg) has a walking speed of ≈4 km/h. Because he concentrates on watching birds he fails to see the large pine tree and hits it full front. He comes to a rapid stop within $\Delta t \approx 0.01$ s. The deceleration is:



$a = \Delta v / \Delta t = 4/0.01 \text{ km/h s} = 1.11 \cdot 10^2 \text{ m/s}^2$

The force during the impact is:
$F = m \cdot a = 82 \text{ kg} \cdot 111 \text{ m/s}^2 = 9111 \text{ N}$
with an approximate impact area of
$A = 5000 \text{ cm}^2$ the pressure is:
$P = F/A = 1.82 \text{ N/cm}^2 = 1.82 \cdot 10^4 \text{ N/m}^2$

If the hiker hits a young pine tree branches will soften the impact:
$\Delta t \approx 0.06$ s
the force F and the pressure P is reduced by a factor of six:
$F = 1.52 \cdot 10^3$ N
$P = 3.04 \cdot 10^3 \text{ N/m}^2$

Falls from great height

The above equation has to be generalized because of energy transfer arguments!

The average force acting on the part of the body which hits the ground is

$F_0 = \mathrm{m} \cdot \frac{v_i}{\Delta t} \cdot \mathrm{q}$

v_i: impact velocity

Δt: collision time

q=1 inelastic stop, no rebounce, all impact energy causes deformation of body or ground

q=2 elastic stop, rebounce, body bounces back with same speed as impact

average deceleration $\bar{a} = v^2/2\Delta h_{CM}$

Body decelerates with average deceleration a from impact velocity *v* to zero while the center of mass of body moves over a distance a/Δ_{CM} during the collision

$$\Rightarrow \quad \Delta t = v/\bar{a} = 2\Delta h_{CM}/v$$

for q=1 (no bounce)

$$F = \frac{-v^2}{s\Delta h_{CM}}$$

with v^2=2g·h h: height of fall

$$F = \mathrm{m}\cdot\mathrm{g}\cdot\ \mathrm{h}/\Delta h_{CM}$$

As larger Δh_{CM}, as smaller the transmitted force

EXAMPLE: Stiffed leg jump on hard ground

mass of the person: m=82 kg

maximum compression of the knee joints (tibia): Δh_{CM}=1cm

F = m·g· h/Δh_{CM} = 82 kg 9.81 m/s^2/1 cm ·h

that yields for a jump from 2m height:

F = 160,884 N

collision time: $\Delta t = 2\Delta h_{CM}/v = 2\Delta h_{CM}/\sqrt{2gh}$

$\Delta t = 3.2 \cdot 10^{-3}$ s

stress (pressure) on each knee joint (average area of joint A≈3.3cm^2)

P = 1/2 F/A = 24,376 N/cm^2 = 2.44·10^8 N/m^2

break point for joints: P≈15,860 N/cm^2

⇒ fracture may occur for stiffed legs jumps from heights of h≥1.5 m!

jumping into water increases Δh_{CM} and Δt:

Δh_{CM} ≈2 m ⇒ F=800 N; P= 242 N/cm^2

$\Delta t \approx 0.64$ s

but belly flop

Δh_{CM} ≈30 cm; A≈1000 cm^2

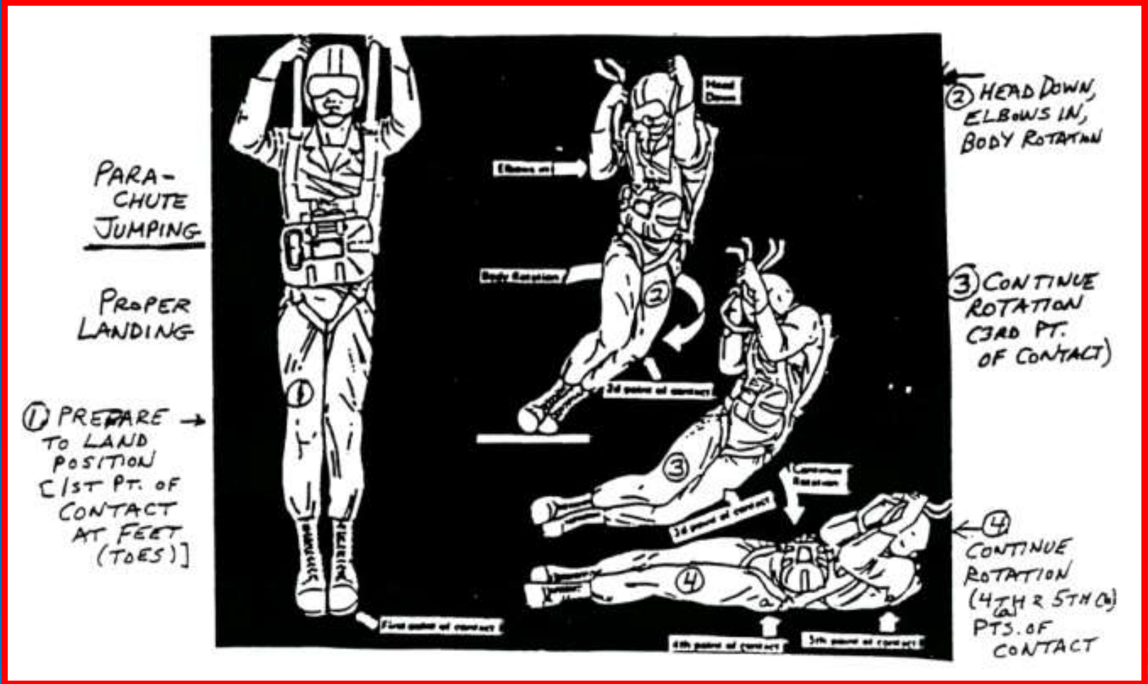
⇒ F = 5333 N; P = 5.33 N/cm^2

Tolerance levels for whole body impact

Threshold for survival: $\approx$ 20 mi/h = 36 km/h = 8.9 m/s

Effects of impact can be reduced by increasing Δt (Δh_{CM}) or by distributing force F over large area to reduce compressing stress.

proper landing techniques for parachutes



To survive a fall the impact pressure should be: $\leq$ 40 lbs/in^2 = 27.6 N/cm^2

For an impact pressure of 35 N/cm^2 $\approx$ 50 % survival chance!

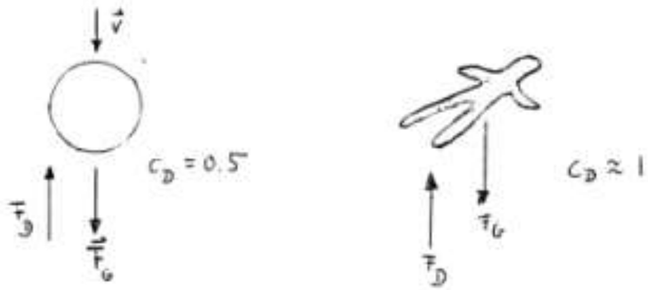
EXAMPLE: Free fall from large heights

for a fall from 300 - 3000 m height the final velocity of a free fall is:

$$v = \sqrt{2gh} = \sqrt{29.81 m/s^2 h} = 77 \text{ - } 243 \text{ m/s}$$

considerably larger than the typically deadly impact velocity

but free fall is idealized case because of decelerating drag and buoyancy forces F_D in the atmosphere. (friction by viscous resistance)



$$m \cdot a = m \cdot g - F_D$$

$$F_D = - \alpha v^2$$

$$\alpha = 1/2\, C_D \cdot \rho \cdot A$$

C_D is the empirical drag coefficient in material of density ρ:

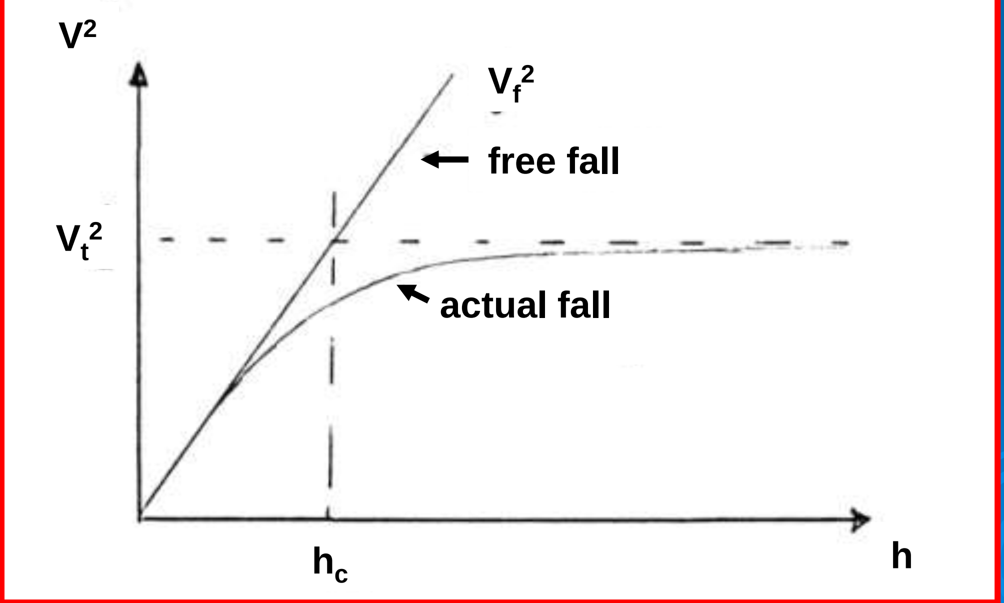
C_D=0.5 for sperical shaped body of area A

C_D ≈1.0 for odd shaped body of area A

Solution of force equation yields final velocity *v*!

$$v = v_t \cdot \sqrt{1 - e^{-h/h_c}}$$

$$v_f \text{ free fall speed } v_f = \sqrt{2gh}$$
$$v_t \text{ terminal velocity } v_t = \sqrt{2gh_c}$$
$$h_c = v_t^2/2\mathrm{g} = \mathrm{m}/(\mathrm{C}_D\rho\mathrm{A})$$



Exponential approach of speed of fall towards the terminal velocity!

Terminal velocity represents the state where the forces are in equilibrium!

h/h_c	e^{-h/h_c}	V/V_t
1	0.37	0.79
2	0.14	0.975
3	0.050	0.975
4	0.018	0.991

speed of free fall levels out at:

$$v_t \approx 120 \text{ mi/h} = 192 \text{ km/h} = 53.3 \text{ m/s}$$

this is only a factor five to six larger than critical survival speed!
if Δt (Δh_{CM}) can be increased survival is possible!

$$F = \text{m}\cdot\text{g}\cdot\frac{h}{\Delta h_{CM}} = \text{m}\frac{v^2}{2\Delta h_{CM}}$$

critical pressure: $P_c = F/\text{A} = \frac{mv^2}{2A\Delta h_{CM}}$

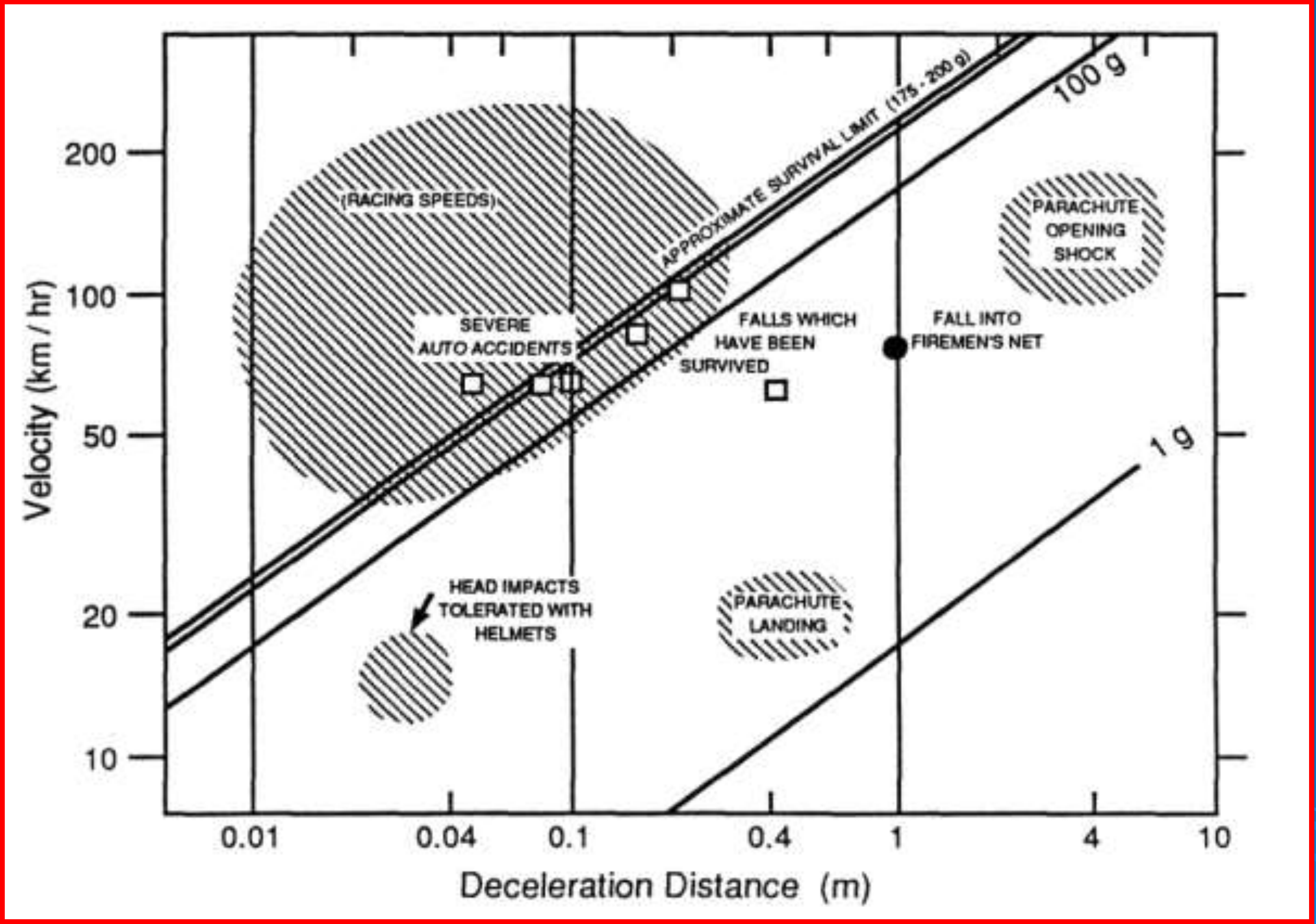
survival possible if $\Delta h_{CM} \geq \frac{mv^2}{2AP_c}$

for $\text{A} \approx 0.34 \text{ m}^2$ (fall on the back) $P_c \approx 50 \text{ lb/in}^2$

$\Delta h_{CM} \geq 1$ m fall into sand or snow

for $P_c \approx 28 \text{ lb/in}^2$ (survival but shock)

$\Delta h_{CM} \geq 2$ m fall into snow, bushes, or water


APPROXIMATE SURVIVAL LIMIT (175 - 200 g)
100 g
1 g
(RACING SPEEDS)
PARACHUTE OPENING SHOCK
SEVERE AUTO ACCIDENTS
FALLS WHICH HAVE BEEN SURVIVED
FALL INTO FIREMEN'S NET
HEAD IMPACTS TOLERATED WITH HELMETS
PARACHUTE LANDING
Velocity (km / hr)
200
100
50
20
10
0.01
0.04
0.1
0.4
1
4
10
Deceleration Distance (m)